

ANNA POMIERNY-WĄSIŃSKA

<https://orcid.org/0000-0002-9957-3698>

The Tadeusz Manteuffel Institute of History
of the Polish Academy of Sciences, Warsaw

MEASURING AND SHAPING THE LATE MEDIEVAL CITY: MATHEMATICAL DESCRIPTIONS OF CITY WALLS IN FLORENCE AND MILAN*

Abstract: The article presents the use of mathematical tools (both calculation and geometry) in the perception of late medieval urban space. For this purpose, two descriptions of city walls of the late medieval Italian city have been compared. The first describes the city walls of Florence, and originates from Giovanni Villani's *Nuova Cronica*; the second pertains to the fortifications of Milan, and originates from the much less known *Cronica extravagans* by Galvano Fiamma. Both authors used data from measurements and knowledge of the principles of geometry to illustrate the splendour of their respective cities and to help readers visualize them.

Keywords: urban form, city walls, late medieval urban chronicles, description of cities, perception of urban space.

I

The characteristics of city walls constituted one of the basic and indispensable elements of medieval descriptions and *laudes civitatum*.¹ Fortifications

* The article was written as part of the project 'Urban Space of Late Medieval Florence: Representations and Perception' (no. 2014/15/N/HS3/01768) financed by the National Science Centre (NCN) of Poland; a much shorter version of this text with some crucial differences was presented at the V Ciclo di Studi Medievali in Florence (3–4 June 2019) and published in the conference proceedings. I would like to thank for detailed suggestions Halina Manikowska and Zofia Anuszkiewicz as well as the anonymous reviewers of this text. I am also very grateful to Andrea Zorzi from University of Florence, the staff of Biblioteca Nazionale di Firenze and Kunsthistorisches Institut in Florenz for their help during my research.

¹ On the *descriptions urbium* and *laudes civitatum* see Ernst Robert Curtius, *European Literature and the Latin Middle Ages*, London, 1953, p. 157; John K. Hyde, 'Medieval Descriptions of Cities', *Bulletin of the John Rylands Library*, 48, 1966, pp. 308–40; Gina Fasoli, *La coscienza civica nei comuni italiani del Duecento*, Todi, 1972, pp. 11–44; Carl Joachim Classen, *Die Stadt im Spiegel der Descriptiones und Laudes Urbium in der antiken und*

played a decisive role in defining the city and its limits.² Their image functioned as a tool to imitate and resemble the Heavenly and Earthly Jerusalem or Rome.³ The fortifications defined the boundaries of the religious community and, together with the nearby sacred places (*loca sacra*), formed its defensive ring (*murus protectionis*). Their course was often marked and commemorated in ritual processions performed to protect the space inhabited by the urban community.⁴ Finally, the walls constituted one of the most important communal goods (*res publica*) characterized by *utilitas publica* — they belonged to the community as a whole and also settled questions of membership in it.⁵

The city gates were not only a link between those *intra* and *extra muros*, but also organized the space located inside the walls, establishing divisions into districts (*portæ*). Their physical location was also inextricably related to the city centre and to the design of the pattern of the principal streets. Due to their numerous functions, walls and gates were one of the most important identifiers of specific places and provided a frame of reference for spatial orientation.

The developing political and financial system in the late medieval Italian city had a crucial role in controlling all activities involved in constructing city walls. The control and supervision covered, among other things, concluding contracts for building individual sections of walls,⁶

mittelalterlichen Literatur bis zum Ende des Zwölften Jahrhunderts, Hildesheim and New York, 1980; Paolo Zanna, 'Descriptiones urbium and Elegy in Latin and Vernaculars, in the Early Middle Ages', *Studi Medievali*, Ser. 3, 32, 1991, pp. 523–96; Elisa Occhipinti, 'Immagini di città: Le laudes civitatum e le rappresentazioni dei centri urbani nell'Italia settentrionale', *Società e storia*, 14, 1991, pp. 23–52; Lucia Nuti, *Cartografie senza le carte: Lo spazio urbano descritto dal Medioevo al Rinascimento*, Milan, 2008, pp. 36–54; 86–90; Paul Oldfield, *Urban Panegyric and the Transformation of the Medieval City, 1000–1300*, Oxford, 2019.

² Lucia Nuti, 'Lo spazio urbano: realtà e rappresentazione', in *Arti e storia nel Medioevo*, ed. Enrico Castelnuovo and Giuseppe Sergi, 4 vols, Turin, 2002–04, vol. 1, pp. 241–82.

³ Chiara Frugoni, *Una lontana città*, Turin, 1983; Bianca Kühnel, *From the Earthly to the Heavenly Jerusalem: Representations of the Holy City in Christian Art of the First Millennium*, Rome, Freiburg and Vienna, 1987; Paul Zumthor, *La Mésure du monde: Représentation de l'espace au Moyen Âge*, Paris, 1993; Keith Lilley, *City and Cosmos: The Medieval World in Urban World*, London, 2009.

⁴ Silvia Mantini, 'Un recinto di identificazione: le mura sacre della città', *Archivio Storico Italiano*, 153, 1995, pp. 211–61.

⁵ Cf. Sara Menzinger, 'Mura e identità civica in Italia e in Francia meridionale', in *Cittadinanze medievali: Dinamiche di appartenenza a un corpo comunitario*, ed. eadem, Rome, 2017, pp. 65–111 (pp. 78–80).

⁶ For the documentation containing contracts for the construction of sections of the third ring of the Florentine walls, including gates, towers and sometimes their sculptural decoration (which was, unfortunately, largely destroyed by the flood of

and making measurements,⁷ which was assigned to surveyors and the master builders who accompanied them.⁸

Building fortifications was also one of the first impulses for the Italian communes to pursue the principle of proportionality in levying special taxes from citizens who owned real estate in the city.⁹ For example, in 1325, during the construction of the outermost ring of walls, which continued for several decades, the Florentine authorities decided to impose a special two-year property tax to help complete the construction of fortifications in one of the districts.¹⁰ The amount depended mainly on whether or not the property owned by a citizen had a façade looking out onto one of the main streets of the city, and on the length of the front wall of the building adjacent to the street. This regulation shows the need for urban space to be rationalized by the use of measurement tools due to the principle of just distribution of responsibilities for the common good. Its effect was to ‘digitalize’ the city space, and to compile measurement records and amounts of tax thus due.¹¹ In the measuring of urban space, the most important variable was the borderline between

1966, and for many years unavailable, but is now restored after conservation work), see Florence, Archivio di Stato (hereinafter ASF), Capitani di Parte Guelfa, Numeri Rossi 103.

⁷ For an earlier example of surveyor’s work in the construction of urban fortifications, although from another region of Europe (Flanders), see Lambert of Ardres, *Historia Comitum Ghisnensium*, ed. Johannes Heller, in MGH SS, vol. 24, Hanover, 1878, pp. 550–642 (p. 642).

⁸ For traces of measurements of the walls carried out in Florence, see ASF, Corporazioni religiose soppresse dal governo francese 119, 607 (1323); 119, 839 (1323); ASF, Corporazioni religiose soppresse da Pietro Leopoldo 508, 374 (1328); cf. Paula Spilner, ‘Ut civitas ampliatur: Studies in Florentine urban development, 1282–1400’, PhD thesis, Columbia University, 1987, p. 208, n. 163; in smaller cities under Florentine rule: ASF, Ufficiali delle Castella 11, c. 11v, 25r, 37r; see also: David Friedman, ‘Who Designed the Plan of the Florentine New Town of San Giovanni Valdarno (1299)’, *Quaderni del Dipartimento Patrimonio Architettonico e Urbanistico*, 37–40, 2009–10, pp. 21–34.

⁹ Regarding Italian and French cities, the most recent work on this subject is: Menzinger, ‘Mura’, pp. 65–109.

¹⁰ ASF, Provvisioni Registri 22, fol. 39v (15 November 1325), see also: *Carteggio inedito d’artisti dei secoli XIV, XV, XVI*, ed. Giovanni Gaye, 3 vols, Florence, 1839–40, vol. 1, p. 468; Franek Sznura, ‘Appunti sull’urbanistica fiorentina tra XIII e XIV secolo’, in *Arnolfo: Alle origini del Rinascimento fiorentino*, ed. Lusanna E. Neri, Florence, 2005, pp. 86–97 (p. 89); for earlier examples of similar regulations in other cities, see Menzinger, ‘Mura’, pp. 67–68.

¹¹ On ‘digitalization’ in the sense of ‘the transformation of forms or images into numerical values’, see Emanuele Lugli, ‘A Mathematical Land: Measurements in Twelfth and Thirteenth-Century Modena and the Po Valley’, in *Was zählt: Ordnungsangebote, Gebrauchsformen und Erfahrungsmodalitäten des ‘numerus’ im Mittelalter*, ed. Moritz Wedell, Köln, 2012, pp. 275–93 (p. 292).

private and public space.¹² These actions resulted in the compilation of ‘coded’ property maps, which not only included spatial information but also categorized it, thereby valorizing the space, its elements and its structures.

In essence, the system for supervising urban construction projects and the search for a fair principle of participation in the financing of public construction projects meant that the language of book-keeping and measurement became tools commonly applied in describing urban space. The fundamental example of its use is found in building regulations.¹³ These regulations were known to the chroniclers, who used documents kept in municipal chancelleries. One should also not underestimate the fact that the two key categories of rationalized description of space — measurement and quantity — had long been present in Roman catalogues and *mirabilia*, as well as in the *descriptions* and *laudes* of other cities in late antiquity and the early Middle Ages.¹⁴ It was also for this reason that late medieval chroniclers, familiar as they were with this convention of describing urban space, so readily embraced and developed a numerical and measurement-based convention in their works.

The subject of the article is the analysis of the use of mathematical tools in the literary descriptions of urban space, conducted in relation to selected passages of two historiographic sources. One text describes the city walls of Florence, and originates from Giovanni Villani’s *Nuova Cronica*, while the other forms part of the much less studied *Cronica extravagans* by Galvano Fiamma and describes the walls of Milan. Both works were written in the 1330s and referred to fortifications built or reconstructed in part during their authors’ lifetimes.¹⁵ Both authors used data from meas-

¹² The most famous examples are *libri terminorum* from Bologna, that is, registers of the boundaries between public and private space at key points of the city, see esp. Jacques Heers, *Espaces publics, espaces privés dans la ville: Le liber terminorum de Bologne (1294)*, Paris, 1984; for more examples from Italian cities: Giampaolo Francesconi and Francesco Salvestrini, ‘La scrittura del confine nell’Italia comunale: Modelli e funzioni’, in *Frontiers in the Middle Ages: Proceedings of the Third European Congress of Medieval Studies*, ed. Outi Merisalo, Louvain-la-Neuve, 2006, pp. 197–221.

¹³ *Statuti urbanistici medievali di Lucca: Gli statuti delle vie e de’ pubblici di Lucca nei secoli XII–XIV: Curia del Fondaco: Statuto 1371*, ed. Domenico Corsi, Venice, 1960; Guido Pampaloni, *Firenze al tempo di Dante: Documenti sull’urbanistica fiorentina*, Rome, 1973; Enrico Guidoni, *Storia dell’urbanistica: Il Duecento*, Rome, 1989, pp. 336–419.

¹⁴ See literature in n. 1, especially the work of Lucia Nuti, which emphasizes the importance of the category of measure, and: *Codice topografico della città di Roma*, ed. Roberto Valentini and Giuseppe Zucchetti, 4 vols, Rome, 1940–53, vol. 3, 1943, pp. 77–110; Cristina Nardella, *Il fascino di Roma nel Medioevo: Le ‘Meraviglie di Roma’ di maestro Gregorio*, Rome, 1997.

¹⁵ Giovanni Villani, *Nuova Cronica*, ed. Giuseppe Porta, 3 vols, Parma, 1990–91

urements of the walls in order to capture the size and magnificence of their cities and by means of these data described the geometric features of urban layouts to make it easier for the readers to visualize them.

II

Giovanni Villani described the outermost ring of the Florentine walls in two chapters of Book 10 of his monumental *Nuova Cronica*.¹⁶ Unfortunately, this description often escapes researchers analysing the use of accounting-related and quantitative tools in chronicles of late medieval cities. Their attention focuses primarily on the great numerical praise of Florence relating to the years 1336–38, where Villani used quantities in his description of the city’s citizens and their educational background, professional groups, food production, buildings in and outside the city, and so on.¹⁷ John Hyde, the author of a seminal study on the descriptions of Italian cities, did not focus on the meaning or character of the description of the walls analysed below; he rather vaguely justified its exclusion from the numerical praise of the city saying that it was placed in an earlier part of the chronicle.¹⁸ I find this explanation insufficient. The two fragments

(hereinafter *Nuova cronica*); *La Cronaca estravagante di Galvano Fiamma*, ed. Sante Ambrogio Céngarle Parisi, Massimiliano David, intro. Paolo Chiesa, Milan, 2013 (hereinafter *Cronica extravagans*).

¹⁶ *Nuova cronica*, 10:256–57.

¹⁷ *Ibid.*, 12:92–94; see Armando Saporì, ‘L’attendibilità di alcune testimonianze cronistiche dell’economia medievale’, in *idem*, *Studi di Storia economica medievale*, Florence, 1940, pp. 127–35; Enrico Fiumi, ‘La demografia fiorentina nelle pagine di Giovanni Villani’, *Archivio Storico Italiano*, 108, 1950, pp. 78–158; *idem*, ‘Economia e vita privata dei fiorentini nelle rilevazioni statistiche di Giovanni Villani’, *Archivio Storico Italiano*, 111, 1953, pp. 207–41; Arsenio Frugoni, ‘G. Villani, Cronica, XI, 94’, *Bullettino dell’Istituto Storico Italiano per il Medioevo e Archivio Muratoriano*, 77, 1965, pp. 229–55; cf. Edwin S. Hunt, *The Medieval Super-Companies: A Study of the Peruzzi Company of Florence*, Cambridge, 1994, pp. 268–71; these chapters of Villani’s chronicles are often compared with the praise of Milan by Bonvesin de la Riva, in which quantity is the fundamental category, Bonvesin de la Riva, *Le meraviglie di Milano*, ed. Paolo Chiesa, Milan, 2009 (hereinafter *De magnalibus Mediolani*).

¹⁸ Cf. Hyde, ‘Medieval Descriptions’, pp. 335–36; the mathematical character of the description of the walls is not mentioned by Alexander Murray, *Reason and Society in the Middle Ages*, Oxford, 1978, pp. 183–86; surprisingly, neither by Mantini, ‘Un recinto’ or Charles T. Davis in ‘Topographical and Historical Propaganda in Early Florentine Chronicles and in Villani’, *Medioevo e rinascimento*, 2, 1988, pp. 33–51; Francesco Salvestrini analyses it only in passing, *idem*, ‘Descrizioni e “laudes” a Firenze nel secolo XIV. Giovanni Villani, la “Florentie urbis et reipublice descriptio”’, Antonio Pucci, Lapo da Castiglionchio, Coluccio Salutati’, in *Antica possessione con belli costumi*, ed. Franek Sznura, Florence, 2005, pp. 205–32 (p. 211). The state of research on Villani’s great description of the walls has significantly changed in recent years owing to David Friedman, who

of the chronicle should not be linked, nor should they be treated as two parts of the same praise of Florence. Nevertheless, both serve to create an encomiastic image of the city and are proof of both authors' adoption of the same (that is, numerical) convention of describing the surroundings. Giovanni Villani was a well-educated merchant;¹⁹ in the 1320s, he often served as a member of the communal committees responsible for tax audits and the management of municipal funds.²⁰ For him, numbers and calculations were the basic tools in understanding the quality of a thing or issue in question, and therefore whenever he could make a reference to a numerical value in his work he tried to do so.²¹ As shown by the numerical praise of the city in Chapters 92–94 of Book 12, he did not refrain from providing his own calculations and estimates; also, when not sure about numbers or measures, he would simply leave a blank.²²

It is worthwhile briefly characterizing the instances where Villani used spatial measures. This will help to specify the areas in which measuring was a useful tool in perceiving space, and a privileged feature of descriptive cartography. The following analysis includes the presence of units in Villani's chronicle (the mile — *miglia*, the cubit — *braccio* or *cubito*, and the measure — *misura*),²³ but it is not my intention to identify his sources of information. Villani used them primarily when he was designing textual cartographies, discussing the location of a place based on the distance from a chosen city (both in Tuscany and in faraway countries in Europe and beyond).²⁴ This use applies primarily to the mile, the largest, supra-regional unit, which had been in use since Roman times.²⁵ In many of these cases, Villani was probably faithful to his sources and their con-

analysed in detail its characteristics and recognized its protocartographic features. His article has been an important inspiration and a basis for the following considerations: 'Urban Design without Maps', in *Arnolfo's Moment*, ed. David FriedmanMargaret Haines and Julian Gardner, Florence, 2009, pp. 161–81.

¹⁹ Ronald G. Witt, 'What Did Giovannino Read and Write? Literacy in Early Renaissance Florence', *I Tatti studies*, 6 1995, pp. 83–114; Franca Ragone, *Giovanni Villani e i suoi continuatori: La scrittura delle cronache a Firenze*, Rome, 1998.

²⁰ For the full curriculum of Villani's offices, see Giovanni Aquilecchia, 'Introduzione', in *Cronica con le continuazioni di Matteo e Filippo*, ed. idem, Turin, 1979, pp. X–XI.

²¹ Murray, *Reason*, p. 184.

²² *Nuova cronica*, 10:256–57; 13:46; cf. Arrigo Castellani, 'Sulla tradizione della Nuova cronica di Giovanni Villani', *Medioevo e rinascimento*, 2, 1988, pp. 53–118 (p. 107).

²³ Angelo Martini, *Manuale di metrologia ossia misure, pesi e monete*, Turin, 1883, pp. 206–11.

²⁴ *Nuova cronica*, 2:1; 5:31; 7:6; 7:8; 7:46; 7:49; 8:7; 8:23; 8:28; 8:37; 8:87; 9:52; 12:26; 12:60; 13:84; 13:102; 13:111.

²⁵ Villani uses the Roman mile when describing the borders of Tuscany: *Nuova cronica*, 2:6; see also Odile Redon, 'Une géographie de la Toscane chez un chroniqueur du XIV^e siècle, Giovanni Villani de Florence', in eadem, *Des forêts et des âmes: Espace et*

ventional geographical descriptions.²⁶ One of the most common situations where this unit was used was in reports from military operations and descriptions of the movements of armies, which required an approximate image of the theatre of war to be portrayed, and the distances covered by troops to be estimated.²⁷ For this type of textual cartography, the category of distance was absolutely fundamental. In addition, it was reminiscent of the literary tradition of the Romans dealing with military campaigns and the art of war. It is also worth noting here that, in this way, Villani used the quintessential tool of travelling merchants to conjure an idea of the world and its geographical description: merchants' itineraries providing information about the distances between successive points in a journey.²⁸

The above comments, however, do not exhaust all occurrences of the mile in *Nuova Cronica*. It is also used in suburban areas as a category for delimiting the legal boundaries that established restrictions and regulated the extent of granted privileges.²⁹ The boundaries were based on the centric principle, whereby the city was the centre of the territory in question; from the centre of the city ran a radius defining the spaces granted protection or privilege. For Villani, the concept of the mile was so important in perceiving and determining the status of the territory around the city that he devoted a separate chapter of his chronicle to the place from which it was traditionally measured.³⁰

Finally, in a few cases, the mile is used when the chronicler specifies the length of the walls and tries to determine the size of the city described by reference to its perimeter. This pertains not only to Florence,³¹ but also to distant³² or ancient cities, biblical and mythical alike.³³ With regard to

société dans la Toscane médiévale, ed. Laurence Moulinier-Brogi, Saint-Denis, 2008, pp. 259–72.

²⁶ On Villani's sources, see Louis Green, *Chronicle into History: An Essay on the Interpretation of History in Florentine Fourteenth-Century Chronicles*, New York, 1972, pp. 155–63; Ragone, *Giovanni Villani*, pp. 13–21.

²⁷ *Nuova cronica*, 8:49; 8:51; 8:70; 8:84; 8:87; 8:102; 8:117; 8:122; 8:124; 8:127; 8:140; 8:145; 9:55; 9:65; 9:78; 9:79; 9:119; 9:46; 9:48; 9:146; 9:175; 9:191; 10:296; 11:8; 11:49; 11:52; 11:153; 11:155; 11:211; 12:28; 12:52; 12:54; 12:64; 12:97; 12:131; 12:132; 12:140; 13:35; 13:59.

²⁸ Cf. Alessandra Debanne, *Lo Compasso de navegare*, Brussels, 2011.

²⁹ *Nuova cronica*, 4:3; 5:1; 6:13. This also applies to the rules of banishment from the city: *ibid.*, 10:219.

³⁰ Villani points at the Florentine *milliarium aureum*, the statue of a lion next to the Arno river flowing through the city, *Nuova cronica*, 5:33.

³¹ *Ibid.*, 10:257.

³² *Ibid.*, 9:58.

³³ See the description of the Tower of Babel, *ibid.*, 1:2; and of the walls of Troy, *ibid.* 1:13.

ancient centres, measuring was a traditional and topical descriptive tool: it helped in gaining an idea of the size of such places (be they fictional or no longer existing) and invested them with a more real character, making it easier to conceptualize a representation of them.

Villani's use of the cubit (*braccio*, *braccio di calimala* — approximately 58.3 cm, the basic unit of measure used in Florence) presents a separate issue. The chronicler uses this particular unit when mentioning the number and height of the towers belonging to the Guelphs and Ghibellines during the struggles between them in the first half of the thirteenth century.³⁴ Then, as the consequence, he also provides the figure of the maximum height of a tower as stipulated by laws established by communal authorities in the mid-thirteenth century. This remark, incidentally, is an example of the use of measuring as a tool of control in Italian cities.³⁵ Passages such as these are particularly significant, emphasizing as they do the vertical dimension of urban space — the height of the buildings and, consequently, the cityscape as seen from afar. Villani himself draws attention to this, when he writes about the supposed restoration of Florence by Charlemagne: 'within the little city there were in a short time more than 150 towers pertaining to citizens, and each one 120 cubits high, without counting those pertaining to the city; and by reason of the height of the many towers which then were in Florence, it is said, that it showed forth from afar as the most beautiful and proudest city of its small size which could be found.'³⁶ Thus, the number of the towers and their height are here the fundamental categories used in shaping the image of the city in the past.

It can be hypothesized that the cubit served Villani to describe space when he tried to create an image of the urban space and its buildings controlled by the commune of Florence. These descriptions include references to buildings or construction projects in Florence and in its outer territories.³⁷ This is due to the sources used by the author, namely the documen-

³⁴ *Ibid.*, 6:9; 7:33; for other example of the height of city buildings, see *ibid.*, 7:5.

³⁵ *Nuova cronica*, 7:39 and *Statuti della repubblica fiorentina*, ed. Giuliano Pinto, Francesco Salvestrini and Andrea Zorzi, 2 vols, Florence, 1999, vol. 2: *Statuto del podestà dell'anno 1325*, 4:41, p. 305; see also John M. Najemy, 'Florentine Politics and Urban Spaces', in *Renaissance Florence: A Social History*, ed. Roger J. Crum, John T. Paoletti, New York, 2006, pp. 19–54 (p. 24); Nuti, *Cartografie*, pp. 69–82; eadem, *Lo spazio urbano*, pp. 260–63.

³⁶ *Nuova cronica*, 4:3; Eng. transl. after: *Villani's Chronicle Being Selections from the First Nine Books of the Croniche Fiorentine of Giovanni Villani*, transl. Rose E. Selfe, ed. Philip H. Wicksteed, London, 1906, p. 67.

³⁷ See also descriptions of the Roman *parlagio* in Florence, which were no longer existent in Villani's time: *Nuova cronica*, 1:36; of a Tuscan aqueduct: *ibid.*, 2:1; of work on the restoration of city bridges: *ibid.*, 13:46.

tation produced by the commune and the legal regulations from which Villani drew information for his chronicle, at times using them as a pretext for describing major urban projects. Measurement and calculation are of fundamental importance in these sources due to the technical and financial nature of their subject matter. It is also important that most of these cases concern projects related to the construction of walls, that is, the elements of public space which enjoyed particular protection under municipal law.³⁸

The cubit also appears several times in the chapter on the great flood of 1333 in Florence.³⁹ There, Villani describes in detail the water levels in various parts of the city; the *braccio* is applied to identify the sections of walls destroyed by the flood and the changes that had taken place in the Arno riverbed. It is possible that some information contained in that chapter was drawn from the documentation drafted by communal officials who estimated the magnitude of damages. The flood of 1333 and its causes were subject to detailed and long debates.⁴⁰ Perhaps because of these heated discussions, Villani tried to make his description as rational and objective as possible with regard to the losses sustained. In addition to that, as was the case with buildings and urban layouts that were difficult to imagine on account of their sheer size, the chronicler undoubtedly referred to measurements for rhetorical effect, and specifically to emphasize the scale of the calamity.

III

The chapters of *Nuova Cronica* devoted to the outermost ring of walls are exceptionally rich in numerical detail compared to all other parts of the work.⁴¹ They were written in the 1330s⁴² and refer to the state of affairs as of 1324, when the chronicler was the official responsible for their

³⁸ Descriptions of erecting Florentine wall rings: *ibid.* 2:1; 10:10; 10:137; 10:256–57; in a city subordinate to Florence: 13:82; cf. David Friedman, ‘The Porta A Faenza and the Last Circle of the Walls of Florence’, in *Essays Presented to Myron P. Gilmore*, ed. Sergio Bertelli and Gloria Ramakus, 2 vols, Florence, 1978, vol. 2, pp. 179–92.

³⁹ *Nuova cronica*, 12:1; similarly in the case of descriptions of other natural disasters: 12:22; 13:123.

⁴⁰ *Ibid.*, 2:2; on the subject of the flooding and accounts about it, see Gerrit J. Schenk, ‘L’alluvione del 1333: Discorsi sopra un disastro naturale nella Firenze medioevale’, *Medioevo e rinascimento*, 21/n.s. 18, 2007, pp. 27–54; Francesco Salvestrini, ‘L’Arno e l’alluvione fiorentina del 1333’, in *Le calamità ambientali nel tardo medioevo europeo*, ed. Michael Matheus et al., Florence, 2010, pp. 231–56.

⁴¹ *Nuova cronica*, 10:256–57.

⁴² See Green, *Chronicle*, pp. 164–69.

construction.⁴³ Villani emphasizes that exercising his official capacity he ordered that detailed measurements of the walls should be made. He also presents the reason for including information from this *mensuratio* in the chronicle, that being to commemorate the magnitude of Florence and present it in an accessible manner to readers from outside the city.⁴⁴

This intention stems from the topical motivation of Villani in writing his chronicle and is also one of the characteristic features of *laudes civitatum*. It is not without significance that in striving to achieve this goal Villani chooses measurement as an aspect of description to facilitate the creation of an image of the city. The references to detailed measurements contained in his work may also betray another of the author's motives, albeit not an openly stated one. In the early 1330s, when serving as the treasurer of the construction of new walls, Giovanni Villani was accused of embezzlement.⁴⁵ The long and detailed description may be related to this accusation as an attempt to counter these allegations and underline the author's competence in his execution of his office.

When analysing the literary representation of space, it is worth noting that Villani introduces elements characteristic of *laudes civitatum* into the description of the Florentine walls, and that he structures these chapters in keeping with the conventions of that literary genre. They fit into the traditionally included part devoted to presenting the location (*locus et situs*) together with remarks concerning the fortifications (*moenium*).⁴⁶ This is also confirmed in the conclusion of the description, where the chronicler marks the transition back to the principal matter of his work.⁴⁷ In the final

⁴³ On *ufficiali delle mura*, numbers, term and competence, see *Statuti della repubblica fiorentina*, vol. 1: *Statuto del capitano del popolo degli anni 1322–1325*, 5:129, pp. 288–90; cf. Spilner, *Ut civitas*, pp. 150–59. The documentation of the construction of the walls mentions Villani as an official only once, just under 1324, see ASF, Capitani di Parte Guelfa, Numeri Rossi 103, fol. 30. The chronicler also mentions the performance of the function when describing earlier events from January 1322: *Nuova cronica*, 10:137. At that time he was one of the *priori*, the highest ranking municipal officials, (15 February 1321–15 February 1322), new decisions regarding the walls were made during this term. For the stages of building the fortifications, see Renzo Manetti, Maria Chiara Pozzana, *Firenze, le porte dell'ultima cerchia di mura*, Florence, 1979, pp. 63–90; Spilner, *Ut civitas*, pp. 116–50.

⁴⁴ *Nuova cronica*, 10:256.

⁴⁵ Researchers associate the crisis of his administrative and political career with this event, see: Aquilecchia, 'Introduzione', pp. X–XI; Hunt, *The Medieval Super-Companies*, p. 269.

⁴⁶ 'De laudibus urbium', in *Rhetores Latini minores*, ed. Carolus Halm, Leipzig, 1863, pp. 587–88.

⁴⁷ 'Lasceremo omai del sito de la cittade di Firenze, ch'assai n'avemo detto, e torneremo a nostra materia', *Nuova cronica*, 10:257.

part one can also find other elements characteristic of the genre. They include selective and conventional enumeration of specific elements of the urban space: in this case, references are made to the bridges on the Arno River⁴⁸ and the *loca sacra* which, according to Villani's account, numbered over a hundred ('not including hospitals, churches and monasteries located near the gates').⁴⁹

The above elements frame the image designed by Villani, although the main part of his description comprises measurements of walls and calculations carried out based upon them.⁵⁰ Two elements — *mensuratio* and *laudatio* — are complementary here and serve each other; *laudatio* defines the framework of the description, while *mensuratio* is its content. The first of two chapters dedicated to the walls is devoted to the characteristics of walls on the northern side of the city (five out of six districts, *sestieri*), and the second to the fortifications of the district south of the river (Oltrarno). We are thus presented with two separate descriptions and measurements, the author himself emphasizing that Oltrarno may have been regarded as a city in its own right.⁵¹

Villani begins the first chapter from the easternmost point: a sixty-cubit-high tower built in the location designated for the construction of the Ponte Reale.⁵² Then, he lists the next twelve sections, which in most cases are marked by gates.⁵³ While determining these points, he also indicates the width and height of the buildings. Thus, the representation of the city provided by Villani is not two-, but three-dimensional. While referring to the sections, the chronicler always specifies not only their extremities, but also the number of towers that formed part of the given section. The *enumeratio* of these sections is listed in a continuous manner and appears to have been copied from the accounting books. The phrases specifying the distance — 'from' (*da la porta*) ... 'to' (*a la porta*) ... 'is/measures' ... (*si ha [misura]*) — are characteristic of *mensurationes*, and there is

⁴⁸ Villani lists four stone bridges, in order from east to west: Rubaconte, Vecchio, Santa Trinita and de la Carraia. The chronicler also mentions the unbuilt, though planned, easternmost Ponte Reale, cf. also *ibid.*, 10:83. Significantly, Villani writes about bridges as permanent elements of space, despite the destructions associated with the 1333 flood; he writes these words, however, probably already after the flood, as evidenced by the mention of the consequences of flooding in the Oltrarno district.

⁴⁹ *Nuova cronica*, 10:257.

⁵⁰ The numerical praise of the city in Book XII is also written into this framework, *ibid.*, 12:94.

⁵¹ *Ibid.*, 10:257.

⁵² On *Ponte Reale* see above n. 48; in the municipal statutes it functions under the name *Pons Popularis*, *Statuti della repubblica fiorentina*, vol. 1, 5:81, p. 261.

⁵³ In two cases, these are large towers in walls, in one case — the Arno river, *Nuova cronica*, 10:256.

no doubt that this part of the description was drawn from a document certifying the *mensuratio*.

In the second part of the chapter, the author summarizes the measurements and makes further calculations. He points out that the dimensions mentioned earlier make it possible to determine the ‘space’ of districts located north of the river.⁵⁴ He sums up all the sections of the walls (7,700 cubits)⁵⁵, gates (nine, including four main gates — *porte mastre* — and five smaller ones known as *postierle*) and towers (forty-five).⁵⁶ He also specifies the distance between the two extremities of the fortifications (4,500 cubits or 1.5 miles), although they did not constitute the city boundary. This remark explains the legitimacy of including this piece of data, as it helps Villani describe the circular shape of the city. He does not use the term ‘diameter’, but this term captures the role of that particular section. It is clear, therefore, that Villani not only uses the language of measure to give an idea of the size of the city, but also uses geometric imagination to verbally represent its form. This is also confirmed by the detailed calculations carried out by David Friedman.⁵⁷ The image conjured up by the chronicler consists in the description of the boundary (the sections of the walls) and his attempt to determine the geometric data defining the form of the city that would correspond to the archetypal circular representation.

The description of fortifications on the southern side of the river from the next chapter (257) follows the same structure: Villani first gives a list of sections and, afterwards, formulates certain conclusions. Some information about this part of the city appears as early as in the first chapter of the description concerned with the length of the section along the river. The chronicler leaves a blank when referring to this section, but he points out that it is shorter than the one on the other bank. This empty space marks a significant difference in the character of the chapters. While the first chapter, concerning the northern part of the city, presents orderly arranged and detailed measurements, the other lacks information on the number of towers and the specified sections are often longer than those north of the river. This may be due to several factors. First of all, Villani himself points out that the construction of the walls in the district of Ol-

⁵⁴ ‘E così troviamo che ’l detto spazio de le cinque sestora de la città di Firenze, a le nuove cerchia di mura, sono [...],’ *ibid.*

⁵⁵ Villani slightly rounds up the final result; according to my calculations, the total length of the sections is 7,696 cubits. Friedman gives a different number of 7,796 cubits. Interestingly, it appears that the figure 7,696 better reflects the calculations carried out by Friedman with the use of Google Earth, emphasizing as they do the accuracy of the measurements provided by Villani, cf. Friedman, ‘Urban Design’, p. 165.

⁵⁶ *Nuova cronica*, 10:256.

⁵⁷ Friedman, ‘Urban Design’, pp. 165–67.

trarno began in 1324: when he was the official responsible for that project and had access to project documentation, the fortifications had not yet been completed. Villani could not, therefore, avail himself of measurements as detailed as those pertaining to the northern part of the city; he could, however, have known the details of the project in progress, such as the approximate length of the planned sections. What we also know is that the task assigned to Villani in his official capacity was to supervise the construction of the walls in the southern part of the city.⁵⁸ The description itself, however, was created or reworked by the author after the great flood of 1333,⁵⁹ when the walls had already been largely completed (although work on the remaining sections and the San Niccolò gate continued for almost two decades).⁶⁰ It should also be noted that the southern walls were built on undulating terrain. Therefore, the hilly character of that district made it difficult to carry out systematic measurements; it was also a much less developed and less urbanized periphery of the city compared to parts adjacent to the walls in the northern part. This observation is also confirmed by the fact that the chronicler describes the first part of the Oltrarno wall in more detail (the section of wall leading from the western part of the city to the Boboli Gardens, where the wall turns towards the hill), mentioning in the final part of the text that the walls at the gate of San Niccolò were unfinished.⁶¹ Villani completes his survey in the place where he began, that is, at the planned but never-built Ponte Reale.

Chapter 257 also includes a summary of the *mensuratio*. Villani provides the number of gates and again tries to determine the width of the Arno River. He leaves a blank space to be filled with the total length of the sections, but gives an approximate calculation in miles (5 miles). However, this list is richer in detail than the one devoted to the northern part of the city since its purpose is to summarize the descriptions of the walls on both sides of the river. It also provides us with more detail about the representation produced by Villani and about the tools he used in the process. Moreover, it is also in this chapter that the chronicler gives the following figures: the width of the moat on either side of the Arno (35 and 30 cubits) and the width of the streets around the fortifications (inside and outside of the wall, 16 cubits); the thickness of the walls (3.5 and 3 cubits) and

⁵⁸ The author emphasizes his own achievements (the erection of the gate and tower at the Camaldoli church); elsewhere, he is critical of the construction of the wall leading to Porta Romana, alluding that blame should be placed on the officials responsible for that building project, *Nuova cronica*, 10:257.

⁵⁹ *Ibid.*

⁶⁰ Manetti, Pozzana, *Firenze*, pp. 89.

⁶¹ *Nuova cronica*, 10:257; see also Friedman, 'Urban Design', p. 165.

their height (20 cubits). Thus, we find here the characteristic, almost topical elements of medieval descriptions of fortifications (*moenium*) — their height and width — which emphasized their *qualitas*. However, the data is much more detailed owing to the personal involvement of the author in the project and to the documentation accessible to him, and his merchantly meticulousness over numbers and measures.

This fragment of the chronicle concludes with two notes concerning the walled enclosure.⁶² Villani uses there the verb *girare* and the nominal derivative *giro*, both referring to the circle. The chronicler uses this term also in his previous descriptions, which underlines the subconscious association of the term with the ring of walls.⁶³ This association functioned on the level of language; when used on its own, void of geometric explanations, it need not have indicated that the city was regular in shape.⁶⁴ It is also worth noting here that the term *giro* may also refer to walking along the walls and carrying out the measurement of city boundaries. Villani reconstructs the route, providing the lengths of the sections divided by towers and gates, crosses the river by the Ponte alla Carraia and walks into the other part of the city; there, his calculations become confused since the hilly terrain is difficult to measure. Finally, he returns to the place where he began. This description is of an extremely practical value, because it illustrates that Villani clearly used the documentation produced by surveyors and that he also availed himself of his own imagination, firmly based on his familiarity with the subject of the description. This ‘experience of space’ is an extremely important element in shaping the representations of urban space.

The figures above present only part of Villani’s calculations. When writing about the walled enclosure, Villani gives the circumference as 14,250 cubits, which he then converts to miles (approximately 5, in keeping with the local convention: 3,000 cubits = 1 mile).⁶⁵ Having determined

⁶² ‘E tanto gira la cittade dentro, cioè le mura senza i fossi e le vie di fuori [...]. E così gira la nostra città di Firenze migliaia XIII, e CCL braccia’, *Nuova cronica*, 10:257; see also Dante Alighieri, *Paradiso*, c. 30:130: ‘vedi nostra città quant’ ella gira.’

⁶³ *Nuova cronica*, 2:1; 4:2; 5:7; 8:145; 9:58; see also the description of Tuscany, *ibid.*, 2:6.

⁶⁴ Cf. Friedman, ‘Urban Design’, pp. 165–66; cf. also anonymous, Latin praise *Florentie urbis descriptio*, written at almost the same time as Villani’s *Nuova Cronica*, where the vernacular term *giro* is used instead of the Latin *circuitus*, ‘Florentie Urbis et Reipublice Descriptio’, in Carl Frey, *Die Loggia dei Lanzi zu Florenz*, Berlin, 1885, pp. 119–123; cf. Salvestrini, ‘Descrizioni’, pp. 212–19. All the quoted cases reinforce the belief that the *giro delle mura* means, above all, the ring or circuit of the walls.

⁶⁵ As noted by Friedman ‘a circle of 14,250 braccia around has a diameter of 4,534 braccia’ (‘Urban Design’, p. 166), which is in line with the conclusions presented in Chap. 256. In the light of Friedman’s calculations, the walled circumference is approximately 14,290 cubits.

the boundary of the city with a measure, he directs his attention towards its interior and the fundamental principles of its layout. He emphasizes that while measuring the walls he also ordered that a measurement be made of the 'cross of the city' (*croce de la detta città*) that was marked by the streets leading through the four most important gates: from east to west (from the Santa Croce gate to the Prato gate, 4,350 cubits) and from north to south (from the gate of San Gallo to the Porta Romana, 5,000 cubits). Thus, Villani introduces yet another basic element in the description of the city's layout by identifying the two axes of the city corresponding to *cardo* and *decumanus* as defined by the Romans.⁶⁶ This is certainly an approximative and conventional approach, as the axes he points out do not consist of equal or consistently oriented sections. This image is complemented by the final element of the city's form: the designation of its centre. The measurements quoted by Villani indicate that it was the Mercato Vecchio, the forum of Florence in the Roman period and the most important economic centre of the city in the Middle Ages. The author gives the dimensions of the sections running from the four gates marking the cross of the city to that central place, which also attests to the permanence of Roman measuring tradition and practice.⁶⁷ Having determined the mathematical relationship between the city centre and its peripheries, Villani also indicates a specific place which in his view is both the point of intersection between the two axes and the centre of the circle of fortifications.⁶⁸

⁶⁶ This is a consequence of systems created by ancient surveyors' practices delimiting the boundaries of newly founded cities on the basis of two main axes, and the close relationship between the manner of presenting the medieval city and Christian cosmology, cf. Oswald A. W. Dilke, 'Maps in the Treatises of Roman Land Surveyors', *The Geographical Journal*, 127, 1961, pp. 417–26; idem, 'Maps in the Service of the State: Roman Cartography to the End of Augustan Era', in *The History of Cartography*, 6 vols, Chicago, 1987–2019, vol. 1, ed. John B. Harley and David Woodward, pp. 201–11; Joseph Rykwert, *The Idea of a Town: The Anthropology of Urban Form in Rome, Italy and the Ancient World*, Cambridge, MA, 1988; for a survey of literature on ancient *agrimensura*, cf. Brian Campbell, *The Writings of the Roman Land Surveyors: Introduction, Text, Translation and Commentary*, London, 2000.

⁶⁷ Villani gives the length of the three equal limbs of the cross (2,200 cubits), and the length of the longer one leading from the Mercato Vecchio to the south (2,800 cubits). Friedman acknowledges 'an artificial symmetry that structures the internal form of the city and by giving a rational dimension to the longer, southern limb they [the figures] also impart a spiritually symbolic character to this particular cross'. It seems to me that this reference to Christian symbolism is an overstatement; in my opinion it is more important to emphasize the importance of creating a geometric image of the city with its geometric centre and regular urban layout, cf. Friedman, 'Urban Design', p. 167.

⁶⁸ 'Il punto della croce e del centro del giro della cittade si ha in su la Calimala, quasi ov'è oggi la casa de' consoli dell'arte de la lana, ch'è tra Calimala e la piazza e loggia d'Orto Sannicchiele.' *Nuova cronica*, 10:257.

This indication marks the pivotal point of his geometrized and rationalized vision of space and appears alongside other elements characteristic of *laudes civitatum*, namely a selective *enumeratio* of elements of space within the walls, with which the description of the location of Florence ends.

The structure of both chapters reflects *mensurationes*, which primarily included the measurement of all sides of the delimited area (usually four sides, but this depended on the terrain, which was never defined by referring to a specific geometrical figure) and, secondly, the calculations of the size of the area in question. In the *mensurationes* related to sales or taxes, spatial measurements were sometimes accompanied by the procedure known as *estimationes*, that is, valuations carried out by surveyors, accountants or other officials specially designated for this purpose. Sources of this kind are rare, but are sometimes preserved in the documentation of individual building projects or in the archives of municipal offices and church institutions.⁶⁹ The exceptions are the *libri terminorum*⁷⁰ and detailed tax documentation.⁷¹ The available sources usually include information that the *mensuratio* had been made and paid for, and the final calculations (the size of the area determined in the measurement unit adopted by the city and the value of a given property). Thus, Villani's text proves to be even more important, enriching as it does our knowledge from a small number of sources.

Retaining the structure of the measurement allowed Villani to supply numerical data and present their interpretation as an idealized image of the city. In the summary of the first chapter, Villani presents Florence built on the diameter of the circle marked by the river flowing through the middle of the city; in the summary of the other chapter, he refers to the image of the city functioning in culture, which was based on a circle, a cross of streets, and a conspicuous centre. This idealization should be treated as a way of perceiving space through geometrization, making it easier to comprehend and refer to known archetypes.⁷² The pieces of information presented in the two chapters

⁶⁹ For examples of *mensurationes* in the vernacular, see ASF, Corporazioni religiose soppresse dal governo francese, serie 86: S. Maria degli Angeli, 64, fols 12r–16r, 17r–30v; ASF, Ospedale di San Matteo, 1, fols 15r–16r, 27r–28v.

⁷⁰ Francesconi, Salvestrini, 'La scrittura del confine'.

⁷¹ On measuring practices of land in the Middle Ages and *mensurationes* performed for the fiscal needs: Arnelle Querrien, 'Techniques et pratiques de la mesure du sol', in *La terre: Connaissance, représentations, mesure au Moyen Âge*, ed. Patrick Gautier Dalché, Turnhout, 2013, pp. 625–72; *De l'estime au cadastre en Europe: Le Moyen Âge*, ed. Albert Rigaudière, Paris, 2006.

⁷² Evgeny A. Zaitsev, 'The Meaning of Early Medieval Geometry: From Euclid and Surveyors' Manuals to Christian Philosophy', *Isis*, 90, 1999, pp. 522–55.

containing the description of walls complement one another in a strikingly evident manner.

The use of written *mensurationes* is attested not only on the level of the text's structure, but also in the terminology used in it. Villani refers several times to the concept of the border (*frontiera*) when writing about a section of city walls. Summarizing the description of the northern part of the city, he also uses the verb *determinare*,⁷³ which should be associated with the Latin *terminare*, defining the basic activity of surveyors, that is, setting boundary points and measuring the distance between them. Together with the description of the city's form, this particular element of the lexical plane of the text confirms the importance of the traditional art of measuring in conceptualizing urban space.

A specific feature of the language used by Villani is the expressions characteristic of exercises in abacus schools, especially in parts in which the chronicler presents a summary of measurements and ensuing calculations.⁷⁴ In the discussed period, the language of mathematical proof was still predominantly descriptive, lacking the symbols and signs known to the modern reader; one of the superior features of that language was its vernacular character. Textbooks of practical mathematics also, and fairly often at that, included examples referring to buildings, that is, the heights of towers, the distances between them, and so on.⁷⁵ Interestingly, however, Villani does not use such terms as 'radius' and 'diameter' when indicating the form of the city.

The measure and geometric characteristics of Villani's description described above should be complemented with comments on issues related to naming and the manner of orientation which enrich his rationalized description of space. The function of boundary marks (*termini*) in this description is undoubtedly assigned to gates and towers. The chronicler pays particular attention to their naming, often providing two names used at the time and their possible origin. These names constitute here the basic elements of urban toponymy. In many cases,

⁷³ 'Aveno diterminata la città di qua dal fiume d'Arno', *Nuova cronica*, 10:256.

⁷⁴ 'E così troviamo'; 'la ritondità de la città e cricuito pigliamo solamente a la latitudine'; 'compitando la detta porta e la detta torre coll'altre.'; 'quando fieno compiute', *ibid.*, 10:256-57.

⁷⁵ The prime example is the pioneering textbooks written by Leonardo da Pisa (known as Fibonacci, active in the first half of the thirteenth century); for a list of Italian abacus treatises, see Warren Van Egmond, *Practical Mathematics in the Italian Renaissance: A Catalog of Italian Abacus Manuscripts and Printed Books to 1600*, Florence, 1981; on teaching practical mathematics, see Robert Black, *Education and Society in Florentine Tuscany, Teachers, Pupils and Schools, c. 1250-1500*, Leiden and Boston, 2007.

while referring to a specific gate, Villani also determines the destination to which the respective road led.

However, this is not the only means of orientation and localization used by Villani. The description of the walls in *Nuova Cronica* is largely innovative in the author's use of the names of winds to determine their course.⁷⁶ What this means is that the list of sections does not consist only of the names and measures, but also information about the point at which the walls turn;⁷⁷ 'with what wind' they run or to what wind they are exposed. Villani does not use the mathematical concept of angle (in the chronicle he does this only with regard to astronomical phenomena),⁷⁸ which is not surprising, since determining angles did not form part of surveying practice. The sides of the delimited area were always determined by surveyors with reference to the cardinal directions, although their names frequently depended on practices adopted in the region.⁷⁹ The use of names of winds seems to be Villani's original idea. That idea also makes it possible to establish a connection between his representations and a portolan chart, in which the Rose of the Winds was used to present a meticulous outline of sea shores to facilitate navigation.⁸⁰ It may seem to the reader of Villani's account that his description of the walls was based on a similar method in

⁷⁶ Cf. Friedman, 'Urban Design', pp. 167–68. The names of winds used by Villani are: *Mezzogiorno*, *Scirocco*, *Levante*, *Tramontana*, *Maestrale*, *Ponente*; some of these names were also used to describe the cardinal directions, see for example, *Nuova cronica*, 2:6.

⁷⁷ Villani uses the words *gombito/gomito* (elbow, colloquially: a turn) or *angolo* (angle).

⁷⁸ *Nuova cronica*, 2:13; 4:1; 12:2; 13:8; 13:41; 13:141. It is worth mentioning that the terminology of winds also played an important role in orientation in the sky; Villani's proficiency in astrology was recently studied by Robert Hand, 'Giovanni Villani and the Great Conjunction of 1345', in *Astrologers and their Clients in Medieval and Early Modern Europe*, ed. Wiebke Deimann and David Juste, Cologne, 2015, pp. 63–82.

⁷⁹ In Florence, these were: *oriens*, *meridies*, *occidens*, *septentrio*; in Milan, they were related to the time of day: *a mane*, *a meridie*, *a sero*, *a monte*. For examples from Florence see: Pampaloni, *Firenze*; for Milan: Paolo Grillo, 'L'introduzione dell'estimo e la politica fiscale del Comune di Milano alla metà del secolo XIII (1240–1260)', in *Politiche finanziarie e fiscali nell'Italia settentrionale (secoli XIII–XV)*, ed. Patrizia Mainoni, Milan, 2001, pp. 26–27. The use of the cardinal directions was a distinct feature of measurement description distinguishing it from the notarial description, where the sides were identified by references to the identity of the owners of the neighbouring property, although it has to be said that the measurement and notarial patterns of description were closely related, see Luciano Lagazzi, *Segni sulla terra: Determinazione dei confini e percezione dello spazio nell'alto Medioevo*, Bologna, 1991.

⁸⁰ Tony Campbell, 'Portolan Charts from the Late Thirteenth Century to 1500', in *The History of Cartography*, vol. 1, pp. 371–463; Patrick Gautier Dalché, *Carte marine et portulan au XIIIe siècle: Le 'Liber de existencia riveriarum et forma maris nostri mediterranei' (Pise, circa 1200)*, Rome, 1995. Victoria Morse, 'The Role of Maps in Later Medieval Society: Twelfth to Fourteenth Century', in *The History of Cartography*, vol. 3, part 1, ed. David Woodward, 2007, pp. 25–52.

shaping the representation of a city and on the use (often only conventional) of the compass.⁸¹ It is also possible that Villani borrowed this method not so much from maps (that is, visual representations of space) as from textual sources or, more specifically, cartographic descriptions in which names of winds were used to determine directions for seafarers.⁸² It is also worth noting that in the Roman *ars ædificatoria* the principles of city location and of the construction of its walls depended on their exposure to winds, and thus their names played an important role in describing the fortifications.⁸³ However, the names of the winds used by the chronicler differ from the ancient ones.

IV

The second description of city walls comes from *Cronica extravagans*, written by a Milanese Dominican, Galvano Fiamma.⁸⁴ John Hyde, mentioned above with reference to Villani's *Nuova Cronica*, described Fiamma, quite unjustly, as 'a nasty plagiarist who enjoyed a reputation on account of his voluminous works',⁸⁵ while other researchers rightly criticized the value of his works because of their propaganda content, which praised the Visconti, who had by the time become the ruling family of Milan.⁸⁶

⁸¹ Villani's contemporary, Opicino de Canistris, also used portolan charts in his visual representations of the world and man, see Karl Whittington, *Body-Worlds: Opicinus de Canistris and the Medieval Cartographic Imagination*, Toronto, 2014. On the compass as a tool used during urban development projects in the period under discussion, see Guidoni, *Storia dell'urbanistica: Il Duecento*, p. 244.

⁸² In such case we would have a description similar to the *collection of written sailing directions*, called *portolano* (not *portolan charts*), T. Campbell, 'Portolan Charts', p. 375; for a discussion on the relationship between *portolano* and *portolan chart*, see *ibid.*, pp. 382–83. For a similar use of the winds, see the description of Europe: *Nuova cronica*, 1:5; analysis of the author's sources in this matter requires a separate study.

⁸³ Vitruvius, *The ten books on architecture*, 6:1–9.

⁸⁴ Sante Ambrogio Céngarle Parisi, 'Introduzione', in *Cronica extravagans*, p. 158. The chronicle is preserved in a single manuscript, from the late fourteenth-century copy by Pietro Ghioldi, the copyist of all Fiamma's secular chronicles: Milan, Biblioteca Ambrosiana, A 275, fols 31r–60v; for more on the copies of Fiamma's works, see Céngarle Parisi, 'Introduzione', pp. 90–110.

⁸⁵ Hyde, 'Medieval descriptions', p. 336.

⁸⁶ Patrick Boucheron calls Fiamma a 'propagandiste infaticable' of the Visconti family, *idem*, *Le Pouvoir de bâtir: Urbanisme et politique éditiale à Milan (XIVe–XVe siècles)*, Rome, 1998, pp. 108–10; see also: Louis Green, 'Galvano Fiamma, Azzone Visconti and the Revival of the Classical Theory of Magnificence', *Journal of the Warburg and Courtauld Institutes*, 53, 1990, pp. 98–113; for recent studies on Fiamma, see: Paolo Tomea, 'Per Galvano Fiamma', *Italia medioevale e umanistica*, 39 1996, pp. 77–120; Massimiliano David, 'La Cronica extravagans de antiquitatibus civitatis Mediolani di Galvano Fiamma: Linee metodologiche per una nuova edizione critica', in *Le cronache*

The title given to the work may be misleading — the work is not so much a chronicle as a praise of the city written in the form of a Latin scholastic treatise; the author himself emphasizes that his method derives from philosophy, not historiography.⁸⁷ This is due to the polemical character of *Extravagans* — the purpose of Fiamma is to defence his previous and highly criticized by contemporaries *Cronica magna de actibus civitatis Mediolani*.⁸⁸ The work consists of *questiones*, in which the author considers subsequent issues from the general history of Milan. The argument is based on the dialectical method, which was used as the best way to conduct a learned dispute. This approach stems also from the intellectual background of the author, who was an educated Dominican: a lector in Dominican *studia* well-versed in theology and canon law, highly knowledgeable in the dialectics of Aristotle and Aquinas, he states in his text that he devoted twenty-five years to reading historiographical works.⁸⁹ We are, therefore, dealing with a radically different author and source; numbers and measures for Fiamma did not constitute such important descriptive elements as they did for Villani. It is therefore even more interesting to study in particular those passages where the Dominican used mathematics to conjure a representation of the urban space of Milan.

Fiamma makes several references to mathematical tools in his reflections on the city, its location and structure. Firstly, he builds a description of the city's location and its size based on the dimensions of the most important complex of public buildings (the Broletto nuovo) and fortifications (*questio X*). Secondly, he uses arithmetic to build a scholastic proof about the sinusoid of the city's history and compare Milan to the largest cities of antiquity (*questio XI*). Thirdly, in the part devoted to urban buildings, he uses detailed measurements of the sections of the wall (*questio XVII*). All these cases involve the use and reworking of the text to which Fiamma refers most often in this work, namely the great praise of Milan by Bonvesin de la Riva of 1288.⁹⁰ In the vast majority of descriptions of buildings and

medievali di Milano, ed. Paolo Chiesa, Milan, 2001, pp. 89–90; Paolo Chiesa, 'Galvano Fiamma fra storiografia e letteratura', in *Courts and Courtly Cultures in Early Modern Europe: Models and Languages*, ed. Simone Albonico and Serena Romano, Roma, 2016, pp. 77–92.

⁸⁷ *Cronica extravagans*, proemio, p. 210.

⁸⁸ The term *extravagantes* was used to define papal decretals which were not included in the *Decretum Gratiani*; here the term reflects the relationship between two works; see *Cronica extravagans*, proemio, p. 210.

⁸⁹ Céngarle Parisi, 'Introduzione', pp. 25–26.

⁹⁰ For more about Bonvesin and his work, see Paolo Chiesa, 'Introduzione', in *De magnalibus Mediolani*, pp. IX–XLIII; Paolo Grillo, 'Il frate, il Popolo e il marchese: Bonvesin da la Riva e la politica milanese', *Archivio Storico Italiano*, 173, 2015, pp. 405–25. For

monuments of the past, the author does not refer to measurements. *Quantitas* is not the basis for describing the *qualitas* of the city in his text, which to him lies more in the *antiquitas* and *romanitas* of Milan, as it is this last that is the dominant element of his text.⁹¹ Fiamma uses measurements only for those spatial elements where the measure-related and geometrical convention of their description hinges on Bonvesin, who based his representation of the city on a numerical catalogue of secular and ecclesiastical buildings and on the characteristics of three main elements: the *curia comunis* (the Broletto nuovo), city walls, and the *forma civitatis*.⁹² These three elements became the foundation of the geometrized image of the city in *Cronica extravagans*, which is the subject of analysis here.

V

The first reference to mathematical tools is found in *questio decima*, in which the Dominican discusses the location of Milan.⁹³ He argues that the city owes its good position to both nature and the art of fortification.⁹⁴ Fiamma describes two rings of walls: the construction of the first, external one with unfinished towers is attributed by the author to Manuel I Komnenos, the Emperor of Constantinople,⁹⁵ whereas the second, internal one is attributed to Azzone Visconti, signor of Milan in 1329–38 (which is for obvious reasons missing from the account by Bonvesin written in 1288).⁹⁶

a detailed study of Bonvesin's and Fiamma's image of urban space, see my forthcoming article in *Archivio Storico Italiano* (2019).

⁹¹ Massimiliano David, 'Galvano Fiamma e la prima antiquaria', in *Cronica extravagans*, pp. 665–82; on the topos of the second Rome in Milan, see Paolo Grillo, 'Una politica della memoria: Milano fra Roma antica, Pavia e Federico Barbarossa', in *La politique de l'histoire en Italie: Arts et pratiques du réemploi, XIVe–XVIIe siècle*, ed. Caroline Callard, Élisabeth Crouzet-Pavan and Alain, Tallon, Paris, 2014, pp. 19–34.

⁹² *De magnalibus Mediolani*, 2:1–12, pp. 24–32; for *forma civitatis*: 'Civitas ipsa orbicularis est ad circuli modum, cuius mirabilis rotunditas perfectionis eius est signum', *ibid.*, 2:4, p. 26.

⁹³ *Cronica extravagans*, Chap. 23–25, p. 258. The *questio* is built in the following order 1) *titulus*; 2) *argumenta*; 3) *sed contra*; 4) *corpus*; 5) *responsiones*; see Céngarle Parisi, 'Introduzione', pp. 172–81.

⁹⁴ *Cronica extravagans*, 23:4; 23:8, p. 260.

⁹⁵ This information does not appear in Bonvesin's text, who writes about an earthen enclosure; the rebuilding of the walls by Manuel I Komnenos was also described by Jacobus da Varagine, who was mentioned several times by the Dominican, see *Iacopo da Varagine e la sua Cronaca di Genova dalle origini al 1297*, ed. Giovanni Monleone, Rome, 1941, p. 353, *Fonti per la Storia d'Italia*, vol. 85, one the datation of the ring see also, Grillo, 'Una politica della memoria', pp. 23–24.

⁹⁶ On the state of Milan's walls see, Aldo A. Settia, 'Cerchie murarie e torri private urbane', in *La costruzione della città comunale italiana, sec. XII–inizio XIV*, Pistoia, 2009,

Similarly to *De magnalibus*, the most important element used to give an idea of the size of the city is the dimensions of its walls. Fiamma claims in the text that the outer ring measures 10,041 cubits, the internal one is 10,045 cubits long, while the width of the moat is 30 cubits.⁹⁷ The characteristics of the fortifications is based on these data, adorned with the topical elements of *laudes civitatum* (for example, the cleanliness of moat water or the quality of fortifications).

At first glance, the 'inaccurate' relationship between the length of the rings may appear striking: the inner circle cannot be larger than the outer one. It raises the question of the insufficient mathematical competence of the author (emphasized by the editor of the source).⁹⁸ Nonetheless, it should be remembered that it may not be particularly helpful to carry out a similar analysis using detailed geometric data and modern mathematical principles (with their characteristic emphasis on extreme precision). This example shows that the principal method of Fiamma's reasoning is to compile collected pieces of information. The Dominican regards them as unassailable and equivalent facts co-creating one and the same reality. All the same, it is nevertheless worth noting that the data provided are not necessarily mutually exclusive. Furthermore, the course of the inner wall, (which depended to a greater extent on the layout of existing buildings and was by necessity irregular) could have been longer than that of the outer ring. Of course, it is quite possible (but not entirely improbable) that the data provided were made up by Fiamma, in accordance with his knowledge and skill. The measurement-based matrix of his description of the city space is borrowed from Bonvesin; Fiamma attempts to complement that matrix with data regarding the new (probably unfinished) ring of walls, so that the image of the city would be complete and consistent. This is how one should treat the 'round' number of towers in the inner ring of the walls given by the author (*cum c. turribus*).

pp. 45–66 (pp. 54–55); Maria T. Donati, 'La cinta muraria milanese', in *Milano e la Lombardia in età comunale: secoli XI–XIII*, Milan, 1992, pp. 150–53; Mario Mirabella Roberti, Antonello Vincenti and Gian Maria Tabarelli, *Milano città fortificata*, Rome, 1983; Gigliola Soldi Rondinini, 'Le fortificazioni urbane medievali', in *Storia illustrata di Milano*, ed. Franco Della Peruta, 10 vols, Milan, 1992–97, vol. 1: *Milano antica e medievale*, pp. 301–20; on city gates, see 'Il caso della Madonna di Riozzo: Le porte medievali della città di Milano', *Arte Lombarda*, 2014, 3: *La Madonna di Riozzo e le sculture delle porte medievali di Milano: [contributi provenienti dal convegno di studi 'Il caso della Madonna di Riozzo' (Castello Sforzesco, 11 novembre 2011)]*.

⁹⁷ The width of the moat here is not 38 as in *De magnalibus*, but 30 cubits; see *De magnalibus Mediolani*, 2:5, p. 26; 1 Milanese cubit = 0.595 m; about Milanese units of measure, see Martini, *Manuale*, p. 350.

⁹⁸ Sante Ambrogio Céngarle Parisi, 'Commento letterario', in *Cronica extravagans*, pp. 429–620 (pp. 472–74).

In the next part of the same *questio*, the author presents the characteristics of the Broletto nuovo.⁹⁹ The Dominican describes the form of the building as a square surrounded by a high wall and located in the city centre.¹⁰⁰ The area of the complex (approximately 10 perches)¹⁰¹ was calculated by means of length and width, as defined by two sections (east to west — 130 cubits, and north to south — 136 cubits).¹⁰² This image emphasizes the symbolic perfection of the city (a quadrilateral located in the centre of a circle).¹⁰³ The language used by Fiamma to describe the form of the Broletto is also worth noting. He uses a mathematical term to describe the size of the area (*plana superficies*), while in describing the centre of the complex he uses the expression *in medio per traversum*.¹⁰⁴ The terms used by Bonvesin are less precise: *spacium* and *in medio*.

It should be noted that the urban layout of Milan in both Bonvesin's and Fiamma's times was based on a grid (consisting of the main streets that connected the gates to the city centre) that was denser than was the archetypal image of two axes, and, as a result, it was impossible to refer to

⁹⁹ A group of public buildings surrounding the square which originally served as the principal meeting place for the citizens (*brolium*). It was the seat of the most important administrative and judicial offices of the commune of Milan; its construction began in 1228; see more Alberto Grimaldi, *Il palazzo della Ragione: I luoghi dell'autorità cittadina nel centro di Milano*, Milan, 1983; Carla Ghisalberti, 'Il Broletto nel quadro dello sviluppo urbano della Milano', *Arte Medievale*, Ser. 2, 3, 1989, pp. 73–83; Paolo Grillo, *Milano in età comunale (1183–1276): Istituzioni, società, economia*, Spoleto, 2001, pp. 56–62; Pierre Racine, 'Les palais publics dans les communes italiennes (XIIe–XIIIe siècles)', in *Le paysage urbain au Moyen-Age*, Lyon, 1981, pp. 133–53; Giancarlo Andenna, 'La delimitazione dello spazio pubblico nelle città: I palazzi dell'impero, dei vescovi e dei comuni', in *Spazio e mobilità nella 'Societas Christiana': Spazio, identità, alterità (secoli X–XIII)*, Milan, 2017, pp. 101–121; Massimiliano David, "'Urbs veneranda nimis': Urbanistica, epigrafia e religione nella rifondazione di Milano, 1171–1233', *Temporis signa*, 10, 2015, pp. 67–84 (pp. 73–78).

¹⁰⁰ 'Broleto est edifitium quadrum, alto muro circumdatum, in medio civitatis fundatum, cuius plana superficies habet circa decem perticam. Est enim ab oriente versus occasum habens .cxxx.cubitos, ab aquilone versus meridiem habet .cxxxvi. cubitos', *Cronica extravagans*, 24:1–2, p. 262.

¹⁰¹ 1 pertica = 654,5179 m².

¹⁰² The result of precise calculation based on these data is 6,364.8 m²; in fact, it is not the sides but the corners of the building that indicated the cardinal points. The description of the building is highly conventionalized and dependent on a measurement pattern, cf. Boucheron, *Le Pouvoir*, fig. 31, p. 544 or David, 'Urbs veneranda nimis', fig. 17, p. 76.

¹⁰³ Lilley, *City and Cosmos*, p. 18. This representation of the city is also emphasized by its topical location in the middle of a plain, between rivers, and the supposed origin of its name (*Mediolanum* = situated in the middle), *De magnalibus Mediolani*, 1:1, p. 16, cf. Oldfield, *Urban Panegyric*, pp. 4–5, 181.

¹⁰⁴ *Cronica extravagans*, 24:4, p. 262.

that image in the text.¹⁰⁵ In order to preserve the model description of space, substitutes of axes appear in the description, that is, the dimensions of the sides of the Broletto. They can be viewed with reference to the two axes of the city described by Villani, as they provide a frame of reference to the cardinal directions, although in a very conventional manner.

Further in his description of the Broletto, the Dominican points to the centre of the complex (the Pallazzo della Ragione) and to the six gates of the complex. The number and names co-determine the spatial organization based on six districts (*portæ*) subordinated to six city gates.¹⁰⁶ Characterizing the individual buildings which make up the complex, Fiamma repeats Bonvesin's model of description based on the cardinal directions. However, his description gives more detail on the buildings of the complex, commenting, for example, on the length of the sides of the Broletto.¹⁰⁷ The description ends with a reference to the city tower and a note that whoever rings its four bells shall have power over the city.¹⁰⁸ The Dominican's goal is to create a representation not only of the central and most important point in the city (*castrum civitatis*), which is compared to ancient citadels¹⁰⁹ but above all of a micro-city symbolizing Milan as a whole. Here, Fiamma consciously uses this trope (a synecdoche), which is a consequence of the semantic relation defined in the text between the complex and the city as a whole: the Broletto means Milan, and Milan means the Broletto.¹¹⁰

The *questio* devoted to the city's location ends with a passage dedicated to the Broletto vecchio, an earlier public complex chosen by the Visconti as their official seat and renewed by Azzone Visconti. Fiamma presents the characteristics of that building, precisely because of its relationship to the ruling family and its particular status of the new

¹⁰⁵ In 1228, it was decided to build eight streets, beginning in the city centre (*broletto*) and running towards six main gates and two posterns, see Grillo, *Milano*, p. 59.

¹⁰⁶ *Cronica extravagans*, 24:1–2, p. 262. Further in the text, Fiamma writes openly about the relationship between the Broletto gates and the gates in the walls: 'Item quelibet porta habet in Broleto nouo portas sibi corespondentes in situ et nomine, et habet spetiale uexillum et certos capitaneos', *Cronica extravagans*, 48:10, p. 304. This interpretation is confirmed by iconographic studies, Maria Laura Gavazzoli Tomea, 'Le pitture duecentesche ritrovate nel Broletto di Milano, documento di un nuovo volgare pittorico nell'Italia padana', *Arte Medievale*, Ser. 2, 4, 1990, pp. 55–69, see also David, 'Urbs veneranda nimis', pp. 74–76.

¹⁰⁷ *Cronica extravagans*, 24:14–15, p. 262.

¹⁰⁸ *Ibid.*, 24:9, p. 262.

¹⁰⁹ 'Posset autem istud Broletum dici castrum ciuitatis, sicut fuit Ylion in Troia et Byrsa in Carthagine. Ylion dicitur Troya et Troya dicitur Ylion; Birsia dicitur Carthago et Carthago dicitur Byrsa', *ibid.*, 24:10, p. 262.

¹¹⁰ 'Sic Broletum dicitur Mediolanum et Mediolanum dicitur Broletum, per figuram synedoce, ubi pars accipitur pro toto', *ibid.*

political centre of Milan. The description of the place, however, are devoid of mathematical categories, which emphasizes the conclusion formulated above: the Dominican referred to these tools only if they were introduced earlier by Bonvesin. The *questio* ends with a closing formula, topical and typical for *laudes civitatum*, to the effect that alongside the buildings mentioned earlier Milan boasts an abundance of palaces, churches and towers which epitomize its size and beauty, and a short verse praising the patronage of Azzone Visconti.¹¹¹

VI

The next *questio* illustrates a convergence of two *rationes*. The first *ratio* is the method of scholastic reasoning, the purpose of which is to learn the truth about the history of the city; the second *ratio* is the use of mathematical operation as a cognitive tool, albeit largely imprecise and simulated. They converge owing to the scholastic form of *expositio*, which allows the author to think creatively, drawing on erudition. The Dominican ponders here the question of whether the city was larger and stronger in the past,¹¹² which invites a comparison of Milan to great ancient cities. The *argumentum* claims that Milan has never been larger; the *responsio* states that in the past its size varied. In the main part of the discussion, the principal category of comparison is measures and numbers drawn from the Dominican's readings. Fiamma decides to use the criterion of measurement as the category of comparison, as it is consistent with the manner used in the previous *questio* based on Bonvesin. His use of comparison as a method is an attempt to formulate an arithmetic calculation to establish the proportion of Milan to the most important cities of antiquity. Firstly, Fiamma determines the size of ancient Milan. From the text known as *Cronica Datiana* he draws information about the width and height of the walls, trying to convert the measures from old to contemporary ones.¹¹³ Then, he gives the number of towers located in the former ring of walls, and decides that on the basis of these three pieces of data it is possible to calculate as many as three unknown dimensions: the height, width and length of the walls (the topical elements of descriptions of medieval *moenia*).¹¹⁴ The reasoning presented in this passage is an imitation of a mathematical proof.¹¹⁵ This is evidenced

¹¹¹ *Ibid.*, 25:6–7, p. 264.

¹¹² *Ibid.*, 26:1, p. 266. The *questio* is built on the same order as *questio decima*.

¹¹³ Fiamma uses the foot 'of a tall man' as a unit of measure with reference to the past and to his day.

¹¹⁴ Oldfield, *Urbran Panegyrics*, pp. 133–37.

¹¹⁵ *Cronica extravagans*, 26:5–8, p. 268.

by the expressions used by the Dominican in the ‘simulated’ arithmetic operation: ‘facta solerti computatione’ or ‘vnde conuenienter computatur [...] ergo fuit in triplo maior.’

Having established the dimensions of ancient Milan, the Dominican presents further arguments: the number of public buildings and their size. In this case, Fiamma refers to the principle of proportionality of the part to the entire urban layout, according to which individual buildings located in the city should be proportionally large with respect to the city as a whole.¹¹⁶ Then, by way of analogy, he applies the presented rule in comparisons with other cities. First, he compares Rome and Milan, beginning with the number of towers in the walls. Then, without trying to provide a calculation, he compiles data from his reading about the fortifications of Carthage, Nineveh and Babylon.

VII

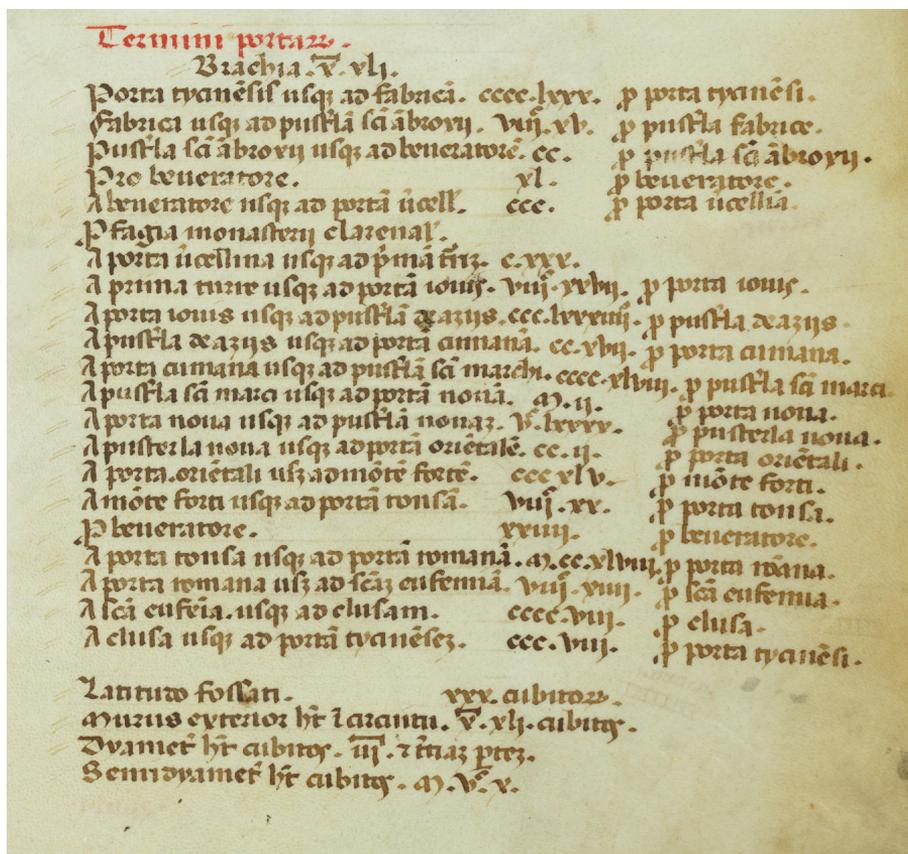
The third and final example of the strong presence of mathematics in *Cronica extravagans* is found in the section on medieval city defences.¹¹⁷ In *questio XVII*, Fiamma presents the characteristics of gates and posterns, discussing their location, the etymology of their names and their architectural features. Right after the general introduction concerning the six main gates, a detailed *mensuratio* of sections of the walls is introduced to the text under the heading *termini portarum* (see ill. 1).¹¹⁸ Its purpose is to bring together and organize information from the descriptions of individual gates and posterns, which, in the light of the text, are the principal element in determining the quality of fortifications.¹¹⁹ This presentation is especially important for our analysis since it uses the same tool of description of the walls as that applied in the discussed chapters of Villani’s *Nuova Cronica*; it also reveals the geometric character of the representation of the city. The fundamental difference between these two measurements is the way they are introduced into the text of chronicle. While Villani decided to describe them in a literary manner in the text of his

¹¹⁶ ‘Hoc etiam probatur ratione quod totum et pars, continens et contentum, debent esse proportionata; sed archus triumphalis, ut dicit cronica Campane, habuit in longitudine tunc temporis ultra duo miliaria, nec umquam fuit per Romanos in toto mundo fabricatus tam magnus archus. Si ergo pars debet proportionari suo toto, et archus, qui pars est ciuitatis, fuit permaximus, ergo ciuitas tunc temporis fuit permaxima’, *Cronica extravagans*, 26:12, p. 268.

¹¹⁷ The characteristics of Roman fortifications, mainly the gates in the walls and the idols located next to them, see *questio XVI*, *ibid.*, 45, 46, pp. 302, 304.

¹¹⁸ *Ibid.*, 48, p. 306; cf. Céngarle Parisi, ‘Commento letterario’, pp. 533–36.

¹¹⁹ *Cronica extravagans*, 56, pp. 314–16.



Ill. 1. *Termini Portarum*, *Cronica extravaganis* of Galvano Fiamma, Milan, Biblioteca Ambrosiana A 275, fol. 46r (detail); the dimensions of the Broletto nuovo are on the next foglio (fol. 47r); between fol. 46r and 47r there is the 'map' of Milan

chronicle, in *Extravaganis* they are incorporated as a list of dimensions imitating the template of an accounting ledger. It is very probable that the list was prepared by Fiamma in the margin as a practical note and later incorporated in the main text by the copyist, Pietro Ghioldi.¹²⁰

The term *terminus* used in the summary statement is a boundary mark (usually a stone or a column). It is one of the basic terms in the vocabulary

¹²⁰ In my opinion there is no doubt that the data were gathered and prepared in form of *termini portarum* by the Dominican (see below); on the characteristics of Fiamma's working process and the copy of Pietro Ghioldi, who made a lot of effort to incorporate the numerous *glosse* of the author in the main text of his works, see Céngarle Parisi, 'Introduzione', pp. 107–09; Chiesa, *Galvano Fiamma fra storiografia e letteratura*, pp. 85–86.

of ancient surveyors (alongside *limes* and *finis*).¹²¹ In the Middle Ages, it was also often used in the general sense as a border. It is worth emphasizing, however, that unlike *limites* and *fines*, *termini* had a material dimension – they were visible and legible points in space.¹²² In this case, the use of the term *termini* underlines the status of gates and posterns as determinants of urban space.

Below the heading '*termini portarum*' only the length of one ring is provided: that of the outer one (10,041 cubits). There follows a list of measurements, which is devoid of any additional comments. The order of the list is based on three columns: the first deals with the sections of the walls, designated in the vast majority by gates and posterns.¹²³ The category of description defining the direction of measurement is characteristic for *mensurationes*, a phrase I have referred to earlier: 'from ... to ...' [(a) ... *usque ad* ...]. There are no additional markers for orienting the representation. The second column contains measurements of the sections they designate. Finally, the third gives information assigning the sections created to a given place (postern or gate) and makes it possible to determine the purpose of the *mensuratio*: it appears that it served to assign the obligations to maintain the sections of fortifications in good condition, those obligations being shared between citizens and ecclesiastical institutions, and resulting from earlier-apportioned responsibilities for gates and posterns.¹²⁴ This allocation of responsibilities may be related to the construction of new fortifications which began during the reign of Azzone Visconti (called 'the inner ring of walls' by Fiamma).

Similar to Villani's description, the *termini portarum* are divided in two parts: detailed measurements of the sides of the delimited space, and a summary of information about the geometrical form. In the latter, the following data can be found: the width of the moat (30 cubits), the

¹²¹ For surveying terminology, see Isidore of Seville, *Etymologiae*, 15:14; B. Campbell, *The Writings*.

¹²² As to the earlier period, see Dieter Werkmüller, 'Recinzioni, confini e segni terminali', in *Simboli e simbolgia nell'Alto Medioevo: Settimane di studio del Centro italiano di studi sull'alto Medioevo*, XXIII, 3-9 aprile 1975, Spoleto, 1975, pp. 641-59; Lagazzi, *Segni*.

¹²³ The only exceptions are abreuvoirs (*beuatoros*) or the so-called 'first tower' (*prima turris*). The phrase *pro faggia monasterio Clarevallensis* in the first column is misplaced, as it belongs to the third column, see Céngarle Parisi, 'Commento letterario', pp. 534-35.

¹²⁴ This conclusion seems to be also confirmed by the analysis of the editors regarding the importance and location of the phrase *pro faggia monasterio Clarevallensis* (see above) and by the content of another work by Fiamma, in which he describes the expenditure of church institutions on the walls, see Federica Cengarle, 'I gruppi scultorei delle porte milanesi: Una forma di comunicazione politica?', *Arte Lombarda*, 2014, 3, pp. 24-29 (p. 25).

perimeter of the outer wall (10,041 cubits), its diameter (3,000 and $\frac{1}{3}$ cubits), the radius (1,510 cubits), and the dimensions of the Broletto nuovo (130 and 136 cubits). Except for the diameter and radius of the circle and the dimensions of the sections of the walls, all data contained in the summary come from *De magnalibus Mediolani*. As was the case with the previously presented mathematical data, Fiamma's calculations are inconsistent: the radius is slightly longer than half the diameter; the measures of individual sections of the walls do not equal the given perimeter. The list of measurements undoubtedly results from a compilation of information from Bonvesin's text and possibly official Milanese documentation, from which the measurements of individual sections were copied (with errors). In Fiamma's work we do not find an explicit glorification of the form of the city, but the mentions of the radius and diameter indicate that he conceptualized the urban space of Milan as a perfect circle.¹²⁵

Another element of the *termini portarum* should be emphasized. In the list there are included also the dimensions of the Broletto nuovo. The complex then is consistently treated as a necessary element of the representation, as it indicates its symbolic and geometric centre. Moreover, the dimensions of the Broletto play here crucial, though extremely conventional function: they help to orientate the city. It is significant, as the geometric data are supplemented with an iconographic representation (see ill. 2). In the literature on the subject devoted to the representations of Milan it was this one that served as the object of analysis.¹²⁶ However, a closer reading

¹²⁵ In his works, Fiamma returned to the dimensions of the city, changed the data and introduced new values. Observation of changes and their confrontation with the chronology of subsequent works allows us to conclude that Fiamma initially relied on the measurements given by Bonvesin and enriched them only with the *mensuratio* of the inner ring of the walls of Azzone Visconti. Over time, however, Fiamma developed a geometric vision of the city, introducing the length of the radius and the diameter of the inner and outer rings into his description. At the last stage, he also tried to calculate the size of the circle designated by the fortifications, see Galvano Fiamma, *Opusculum de rebus gestis ab Azone, Luchino et Johanne Vicecomitibus ab anno MCCCXXVIII usque ad annum MCCCXLII*, ed. Carlo Castiglioni, Bologna, 1938, 17:8–9, p. 26, *Rerum Italicarum Scriptores*, n.s., vol. 12, part 4; Cénarle Parisi, 'Commento letterario', p. 473, tab. 1.

¹²⁶ Ettore Verga, *Catalogo ragionato della Raccolta cartografica e saggio storico sulla cartografia milanese*, Milan, 1911, pp. 3–11; Lucio Gambi, Maria Christina Gozzoli, *Le città nella storia d'Italia: Milano*, Rome and Bari, 1982, pp. 9–12; Massimiliano David and Pierluigi Tozzi, 'Opicino de Canistris e Galvano Fiamma: l'immagine della città e del territorio nel Trecento lombardo', in *La pittura in Lombardia: Il Trecento*, ed. Valerio Terraroli, Milan, 1992, pp. 339–61; Massimiliano David, 'Un esempio trecentesco di cartografia antiquaria in Galvano Fiamma', *Geografia antiqua*, 2, 1993, pp. 123–32; Patrick Boucheron, 'La carta di Milano di Galvano Fiamma-Piero Ghioldi (fine XIV secolo)', in *Rappresentare la città: Topografie urbane nell'Italia di antico regime*, ed. Marco Folin, Reggio Emilia, 2010, pp. 77–97; Silvia Beltramo, 'Describing and "Mapping the Town" Using Iconographic



Ill. 2. The 'map' of Milan, *Cronica extravagans* of Galvano Fiamma, Milan, Biblioteca Ambrosiana A 275, fol. 46v

of *Extravagans* makes it possible to emphasize the complementary character of these three elements: (i) the literary description; (ii) the list of the dimensions of the city; (iii) the specific ‘map’ of the city.¹²⁷ I use the term ‘map’ only very tentatively, as it is anachronistic in this context.¹²⁸ We are dealing here with a hybrid representation, where several different ways of representing a city are used.¹²⁹ At first glance, the representation resembles medieval cosmological diagrams.¹³⁰ One can see there two perfect circles centred on the perfect square of the Broletto:¹³¹ the first represents the Roman walls,¹³² while the other represents the walls of medieval Milan. The gates are marked on both circles and the lengths of the sections from the *mensuratio* are also provided, though only on the second one. The representation also contains other elements: the cardinal points, important religious buildings, cities, rivers, and bridges. Geographic and topographical knowledge of Milan and its dependencies is laid out on a symbolic, highly geometrical structure. Therefore, the ‘map’ visually synthesizes the knowledge derived from the descriptions contained in *Extravagans*. The measurement-based description of the city provided in the *termini portarum* constitute the fundamental principle of the visual representation of the city and strengthen the power of the representation.¹³³

and Literary Sources: Cities in the Late Middle Ages in Italy’, in *Portraits of the City: Representing Urban Space in Later Medieval and Early Modern Europe*, ed. Katrien Lichtert, Jan Dumolyn and Maximilian P.J. Martens, Turnhout, 2014, pp. 143–158 (pp. 155–56).

¹²⁷ For a detailed comparison of the *termini portarum* and the ‘map’, see Massimiliano David, ‘Mappa di Milano con le mura romane e comunali’, in *Cronica extravagans*, pp. 716–20.

¹²⁸ Patrick Gautier Dalché, ‘Maps, Travel and Exploration in the Middle Ages: Some Reflections about Anachronism’, *The Historical Review*, 12, 2015, pp. 143–62.

¹²⁹ Cf. Morse, *The Role of Maps*, pp. 31–32; Patrick Gautier Dalché, ‘Un problème d’histoire culturelle: Perception et représentation de l’espace au Moyen Age’, *Médiévales*, 18, 1990, pp. 5–15.

¹³⁰ On the diagram as a way of presenting urban space in Italy, see Imke Wartenberg, *Bilder der Rechtsprechung: Spätmittelalterliche Wandmalereien in Regierungsräumen italienischer Kommunen*, Berlin, 2015, pp. 120–27.

¹³¹ The Broletto is incorrectly oriented according to the convention adopted by Bonvesin and imitated by Fiamma.

¹³² The descriptions of Roman walls of Fiamma do not contain measurement information. In *Extravagans*, the Dominican describes the Roman gates, but does not describe the ring itself, *Cronica extravagans*, 45; 47, pp. 302; 304.

¹³³ Cf. Gautier Dalché, ‘Maps’, p. 153.

Conclusions

The works of Villani and Fiamma are difficult to compare. These two authors differ in terms of their intellectual background and literary temperament. However, a close reading of the passages from their works devoted to the characteristics of city walls allows us to formulate interesting conclusions concerning the mathematical approach to describing urban space.

Both in *Nuova Cronica* and in *Cronica extravagans*, the descriptions of fortifications are an essential element in presenting the city's location. Villani and Fiamma decide to introduce survey measurements into their descriptions, imitating *mensurationes* and *terminationes*. Their goal is thus to define the city's boundary and make an attempt at presenting an orderly urban layout and the complementary character of the relations between the city centre (indicated strongly in both texts) and its peripheries. All this not only indicates that their ways of defining space were heavily influenced by the practices of surveying, but also that the authors were focused on the form of the city. It seems reasonable to hypothesize that such a far-reaching application of mathematical tools in a literary description is a *signum temporis* of chroniclers' day. The description of the city that is contemporary to them is a highly geometricized and *rationalized* one.

Giovanni Villani uses his own experience and knowledge gained from his official role in the construction of the walls, he also adopt official survey documentation with great skill and displays his knowledge of geometry. Galvano Fiamma, on the other hand, builds his representation on the basis of *De magnalibus Mediolani*, although his description is not a mere copy of the data provided by his predecessor, but an interpretation. The Dominican develops the mathematical discourse of Bonvesin in order to expand a geometric visualization. The list of *termini portarum* compiled by Fiamma is probably a creative attempt to provide complete measurements of the city walls based on various sources and his own imagination, and is connected to the great project of his friend and *signore*, Azzone Visconti.

Finally, it is worth adding that these are not the only descriptions of the walls of Milan and Florence provided by these authors.¹³⁴ The remain-

¹³⁴ For Villani see the description of the previously built ring of walls of Florence, *Nuova cronica*, 4:2, where the dominant feature is an attempt to refer Florence to Rome using the geography of the holy places, Mantini, 'Un recinto', pp. 239–41; cf. Davis, 'Topographical and historical propaganda', pp. 33–51; Emiliano Scampoli, *Firenze: Archeologia di una città*, Florence, 2010, pp. 234–39; for Fiamma, see n. 131 and David, 'Galvano Fiamma e la prima antiquaria'; Paolo Chiesa, 'La Milano romana di Galvano

ing ones, however, concern rings from earlier times. I believe that this is because of the lack of narrative tools for describing the fortifications built during the lifetime of chroniclers (at least in case of Florence¹³⁵). The representations of walls constructed at the time of writing were for obvious reasons lacking in meanings and narratives (for example, those related to the *loca sacra* or underlining the *romanitas* of the city). The authors viewed the new projects as redefining the urban space and its perception and, consequently, searched for other tools to present this redefinition. It seems to me that this was one of the reasons for choosing to apply mathematical discourse, that allowed them to refer the urban space they described also to the traditional representation, which was fundamental in attempts to conceptualize and rationalize the world. Villani and Fiamma go to great lengths to impart a circular shape to their representations, because geometrization allows their microworlds to be presented as being in accordance with a model of the macrocosm. A similar pursuit is described by Dante in the closing stanzas of the *Divine Comedy*:

As the geometer who sets himself
To square the circle and who cannot find,
For all his thought, the principle he needs,
Just so was I on seeing this new vision
I wanted to see how our image fuses
Into the circle and finds its place in it.¹³⁶

Summary

The article presents the phenomenon of the rationalization of urban space, more specifically, the mathematical tools (both calculation and geometry) used in the perception of late medieval urban space. For this purpose, two chronicles have been compared: the famous and well-known Giovanni Villani's *Nuova Cronica* (Florence) and the much less known and less widely studied *Cronica extravagans*

Fiamma', in *Milano e la chiesa di Milano prima di Ambrogio*, ed. Raffaele Passarella, Milano, 2018, pp. 221–38.

¹³⁵ The last ring of Florentine fortifications, which construction observed Villani, changed drastically the size of the city and, consequently, the whole system of perception of its peripheries. The project launched by Azzone Visconti in Milan was not so spectacular; the 'inner' walls were actually close to the previous earth ramparts and consequently they could not change so radically the organization of urban space and the 'system' of urban spatial narratives. Nevertheless, in my opinion, the project of new walls and its documentation should be treated as one of the reasons for choosing this kind of description by Fiamma.

¹³⁶ Dante Alighieri, *Paradise*, transl. James Finn Cotter, c. 33:133–38.

by Galvano Fiamma (Milan). Both texts were written in this same epoch (in the 1330s); Villani and Fiamma differed significantly as regard to their intellectual background, profession and chronicler's temperament. However, both authors used their knowledge of the principles of geometry to illustrate the splendour of their respective cities and to help readers visualize its urban forms. They created a similar image of the city, which stems from a long-standing archetypal image of the city based on the figure of circle. The study addresses the following questions: What mathematical tools (accounting-related and geometric) were used to conceptualize space? On what occasion did the chroniclers use them? What role did they play in their descriptions of space? The main attention is paid to the descriptions of city walls, but the article characterizes also the other instances where the chroniclers used spatial measures and mathematical tools. This helps to specify the areas in which mathematical rationalization was a useful tool in perceiving space, and a privileged feature of descriptive cartography.

Bibliography

- Andenna, Giancarlo, 'La delimitazione dello spazio pubblico nelle città: I palazzi dell'impero, dei vescovi e dei comuni', in *Spazio e mobilità nella 'Societas Christiana': Spazio, identità, alterità (secoli X-XIII)*, Milan: Vita e pensiero, 2017, pp. 101–21.
- Beltramo, Silvia, 'Describing and "Mapping the Town" Using Iconographic and Literary Sources: Cities in the Late Middle Ages in Italy', in *Portraits of the city: Representing urban space in later medieval and early modern Europe*, ed. Katrien Lichtert, Jan Dumolyn and Maximiliaan P.J. Martens, Turnhout: Brepols, 2014, pp. 143–58, *Studies in European Urban History (1100–1800)*, vol. 31.
- Black, Robert, *Education and Society in Florentine Tuscany, Teachers, Pupils and Schools, c. 1250–1500*, Leiden and Boston: Brill, 2007.
- Bonvesin de la Riva, *Le meraviglie di Milano*, ed. Paolo Chiesa, Milan: Mondadori, 2009.
- Boucheron, Patrick, 'La carta di Milano di Galvano Fiamma-Piero Ghioldi (fine XIV secolo)', in *Rappresentare la città: Topografie urbane nell'Italia di antico regime*, ed. Marco Folin, Reggio Emilia: Diabasis, 2010, pp. 77–97.
- Boucheron, Patrick, *Le Pouvoir de bâtir: Urbanisme et politique éditairiale à Milan (XIVe-XVe siècles)*, Rome: École française de Rome, 1998.
- Campbell, Brian, *The Writings of the Roman Land Surveyors: Introduction, Text, Translation and Commentary*, London: Society of the Promotion of Roman Studies, 2000.
- Campbell, Tony, 'Portolan Charts from the Late Thirteenth Century to 1500', in *The History of Cartography*, 6 vols, Chicago: University of Chicago Press, 1987–2019, vol. 1, ed. John B. Harley and David Woodward, pp. 371–463.
- Chiesa, Paolo, 'Galvano Fiamma fra storiografia e letteratura', in *Courts and Courtly Cultures in Early Modern Europe: Models and Languages*, ed. Simone Albonico and Serena Romano, Roma: Viella, 2016, pp. 77–92.

- Chiesa, Paolo, 'La Milano romana di Galvano Fiamma', in *Milano e la chiesa di Milano prima di Ambrogio*, ed. Raffaele Passarella, Milan: Biblioteca Ambrosiana, 2018, pp. 221–38.
- Classen, Carl Joachim, *Die Stadt im Spiegel der Descriptiones und Laudes Urbium in der antiken und mittelalterlichen Literatur bis zum Ende des Zwoelften Jahrhunderts*, Hildesheim and New York: Georg Olms, 1980.
- La Cronaca estravagante di Galvano Fiamma*, ed. Sante Ambrogio Céngarle Parisi, Massimiliano David, introd. Paolo Chiesa, Milan: Casa del Manzoni, 2013.
- Le cronache medievali di Milano*, ed. Paolo Chiesa, Milan: Vita e Pensiero, 2001.
- Curtius, Ernst Robert, *European Literature and the Latin Middle Ages*, London: Princeton University Press, 1953.
- David, Massimiliano, 'Un esempio trecentesco di cartografia antiquaria in Galvano Fiamma', *Geografia antiqua*, 2, 1993, pp. 123–32.
- David, Massimiliano, "'Urbs veneranda nimis": Urbanistica, epigrafia e religione nella rifondazione di Milano, 1171–1233', *Temporis signa*, 10, 2015, pp. 67–84.
- David, Massimiliano, Pierluigi Tozzi, 'Opicino de Canistris e Galvano Fiamma: l'immagine della città e del territorio nel Trecento lombardo', in *La pittura in Lombardia: Il Trecento*, ed. Valerio Terraroli, Milan: Electa, 1992, pp. 339–61.
- Davis, Charles T., 'Topographical and Historical Propaganda in Early Florentine Chronicles and in Villani', *Medioevo e rinascimento*, 2, 1988, pp. 33–51.
- Dilke, Oswald A.W., 'Maps in the Service of the State: Roman Cartography to the End of Augustan Era', in *The History of Cartography*, 6 vols, Chicago: Chicago University Press, 1987–2019, vol. 1, ed. John B. Harley and David Woodward, pp. 201–11.
- Dilke, Oswald A.W., 'Maps in the Treatises of Roman Land Surveyors', *The Geographical Journal*, 127, 1961, pp. 417–26.
- Donati, Maria T., 'La cinta muraria milanese', in *Milano e la Lombardia in età comunale: secoli XI–XIII*, Milan: Silvana Editoriale, 1992, pp. 150–53.
- Fasoli, Gina, *La coscienza civica nei comuni italiani del Duecento*, Todi: Accademia Tuderina, 1972.
- Fiumi, Enrico, 'La demografia fiorentina nelle pagine di Giovanni Villani', *Archivio Storico Italiano*, 108, 1950, pp. 78–158.
- Fiumi, Enrico, 'Economia e vita privata dei fiorentini nelle rilevazioni statistiche di Giovanni Villani', *Archivio Storico Italiano*, 111, 1953, pp. 207–41.
- Francesconi, Giampaolo, Francesco Salvestrini, 'La scrittura del confine nell'Italia comunale: Modelli e funzioni', in *Frontiers in the Middle Ages: Proceedings of the Third European Congress of Medieval Studies*, ed. Outi Merisalo, Louvain-la-Neuve: Fédération Internationale des Instituts d'Études Médiévales, 2006, pp. 197–221.
- Frey, Carl, *Die Loggia dei Lanzi zu Florenz*, Berlin: Hertz, 1885.
- Friedman, David, 'Who Designed the Plan of the Florentine New Town of San Giovanni Valdarno (1299)', *Quaderni del Dipartimento Patrimonio Architettonico e Urbanistico*, 37–40, 2009–10, pp. 21–34.

- Friedman, David, 'Urban Design without Maps', in *Arnolfo's Moment*, ed. David Friedman, Margaret Haines and Julian Gardner, Florence: Olschi, 2009, pp. 161–81.
- Frugoni, Arsenio, 'G. Villani, Cronica, XI, 94', *Bullettino dell'Istituto Storico Italiano per il Medioevo e Archivio Muratoriano*, 77, 1965, pp. 229–55.
- Gautier Dalché, Patrick, *Carte marine et portulan au XIIe siècle: Le 'Liber de existencia riveriarum et forma maris nostri mediterranei' (Pise, circa 1200)*, Rome: École française de Rome, 1995.
- Ghisalberti, Carla, 'Il Broletto nel quadro dello sviluppo urbano della Milano', *Arte Medievale*, Ser. 2, 3, 1989, pp. 73–83.
- Green, Louis, 'Galvano Fiamma, Azzone Visconti and the Revival of the Classical Theory of Magnificence', *Journal of the Warburg and Courtauld Institutes*, 53, 1990, pp. 98–113.
- Grillo, Paolo, 'Il frate, il Popolo e il marchese: Bonvesin da la Riva e la politica milanese', *Archivio Storico Italiano*, 173, 2015, pp. 405–25.
- Grillo, Paolo, *Milano in età comunale (1183–1276): Istituzioni, società, economia*, Spoleto: Centro italiano di studi sull'alto Medioevo, 2001.
- Grillo, Paolo, 'Una politica della memoria: Milano fra Roma antica, Pavia e Federico Barbarossa', in *La politique de l'histoire en Italie: Arts et pratiques du réemploi, XIVe–XVIIe siècle*, ed. Caroline Callard, Élisabeth Crouzet-Pavan and Alain Tallon, Paris: PU Paris-Sorbonne, 2014, pp. 19–34.
- Grimaldi, Alberto, *il palazzo della Ragione: I luoghi dell'autorità cittadina nel centro di Milano*, Milan: Arcadia, 1983.
- Heers, Jacques, *Espaces publics, espaces privés dans la ville: Le liber terminorum de Bologne (1294)*, Paris: Éditions du CNRS, 1984.
- Hunt, Edwin S., *The Medieval Super-Companies: A Study of the Peruzzi Company of Florence*, Cambridge: Cambridge University Press, 1994.
- Hyde, John K. 'Medieval Descriptions of Cities', *Bulletin of the John Rylands Library*, 48, 1966, pp. 308–40.
- Lagazzi, Luciano, *Segni sulla terra: Determinazione dei confini e percezione dello spazio nell'alto Medioevo*, Bologna: CLUEB, 1991.
- Lilley, Keith, *City and Cosmos: The Medieval World in Urban World*, London: Reaktion Books, 2009.
- Lugli, Emanuele, 'A Mathematical Land: Measurements in Twelfth and Thirteenth-Century Modena and the Po Valley', in *Was zählt: Ordnungsangebote, Gebrauchsformen und Erfahrungsmodalitäten des 'numerus' im Mittelalter*, ed. Moritz Wedell, Köln: Böhlau, 2012, pp. 273–93.
- Manetti, Renzo, Maria Chiara Pozzana, *Firenze, le porte dell'ultima cerchia di mura*, Florence: CLUSF, 1979.
- Mantini, Silvia, 'Un recinto di identificazione: le mura sacre della città', *Archivio Storico Italiano*, 153, 1995, pp. 211–61.
- Menzinger, Sara, 'Mura e identità civica in Italia e in Francia meridionale', in *Cittadinanze medievali: Dinamiche di appartenenza a un corpo comunitario*, ed. Sara Menzinger, Rome: Viella, 2017, pp. 65–111.

- Morse, Victoria, 'The Role of Maps in Later Medieval Society: Twelfth to Fourteenth Century', in *The History of Cartography*, 6 vols, Chicago: Chicago University Press, 1987–2019, vol. 3, part 1, ed. David Woodward, 2007, pp. 25–52.
- Murray, Alexander, *Reason and Society in the Middle Ages*, Oxford: Clarendon Press, 1978.
- Nardella, Cristina, *Il fascino di Roma nel Medioevo: Le 'Meraviglie di Roma' di maestro Gregorio*, Rome: Viella, 1997.
- Nuti, Lucia, *Cartografie senza le carte: Lo spazio urbano descritto dal Medioevo al Rinascimento*, Milan: Jaca Book, 2008.
- Nuti, Lucia, 'Lo spazio urbano: realtà e rappresentazione', in *Arti e storia nel Medioevo*, ed. Enrico Castelnuovo and Giuseppe Sergi, 4 vols, Turin: Einaudi, 2002–04, vol. 1, pp. 241–82.
- Occhipinti, Elisa, 'Immagini di città: Le *laudes civitatum* e le rappresentazioni dei centri urbani nell'Italia settentrionale', *Società e storia*, 14, 1991, pp. 23–52.
- Oldfield, Paul, *Urban Panegyric and the Transformation of the Medieval City, 1000–1300*, Oxford: Oxford University Press, 2019.
- Racine, Pierre, 'Les palais publics dans les communes italiennes (XIIe–XIIIe siècles)', in *Le paysage urbain au Moyen-Age*, Lyon: Presses Universitaires de Lyon, 1981, pp. 133–53.
- Ragone, Franca, *Giovanni Villani e i suoi continuatori: La scrittura delle cronache a Firenze*, Rome: Istituto Storico Italiano per il Medioevo, 1998.
- Redon, Odile, *Des forêts et des âmes: Espace et société dans la Toscane médiévale*, ed. Laurence Moulinier-Broggi, Saint-Denis: Presses Universitaires de Vincennes, 2008.
- Rykwert, Joseph, *The Idea of a Town: The Anthropology of Urban Form in Rome, Italy and the Ancient World*, Cambridge, MA: M.I.T. Press, 1988.
- Salvestrini, Francesco, 'L'Arno e l'alluvione fiorentina del 1333', in *Le calamità ambientali nel tardo medioevo europeo*, ed. Michael Matheus et al., Florence: Firenze University Press, 2010, pp. 231–56.
- Salvestrini, Francesco, 'Descrizioni e "laudes" a Firenze nel secolo XIV. Giovanni Villani, la "Florentie urbis et reipublice descriptio"', Antonio Pucci, Lapo da Castiglionchio, Coluccio Salutati', in *Antica possessione con belli costumi*, ed. Franek Sznura, Florence: Aska, 2005, pp. 205–32.
- Sapori, Armando, 'L'attendibilità di alcune testimonianze cronistiche dell'economia medievale', in Armando Sapori, *Studi di Storia economica medievale*, Florence: Sansoni, 1940, pp. 127–35.
- Schenk, Gerrit J., 'L'alluvione del 1333: Discorsi sopra un disastro naturale nella Firenze medievale', *Medioevo e rinascimento*, 21/n.s. 18, 2007, pp. 27–54.
- Settia, Aldo A., 'Cerchie murarie e torri private urbane', in *La costruzione della città comunale italiana, sec. XII-inizio XIV*, Pistoia: Centro Italiano di Studi di Storia e d'Arte, 2009, pp. 45–66.
- Soldi Rondinini, Gigliola, 'Le fortificazioni urbane medievali', in *Storia illustrata di Milano*, ed. Franco Della Peruta, 10 vols, Milan: Sellino, 1992–97, vol. 1: *Milano antica e medievale*, pp. 301–20.

- Statuti della repubblica fiorentina*, ed. Giuliano Pinto, Francesco Salvestrini and Andrea Zorzi, 2 vols, Florence: Olschi, 1999.
- Sznura, Franek, 'Appunti sull'urbanistica fiorentina tra XIII e XIV secolo', in *Arnolfo: Alle origini del Rinascimento fiorentino*, ed. Lusanna E. Neri, Florence: Pagnai Polistampa, 2005, pp. 87–97.
- La terre: Connaissance, représentations, mesure au Moyen Âge*, ed. Patrick Gautier Dalché, Turnhout: Brepols, 2013.
- Tomea, Paolo, 'Per Galvano Fiamma', *Italia medioevale e umanistica*, 39, 1996, pp. 77–120.
- Van Egmond, Warren, *Practical Mathematics in the Italian Renaissance: A Catalog of Italian Abacus Manuscripts and Printed Books to 1600*, Florence: Istituto e Museo di Storia della Scienza, 1981.
- Villani, Giovanni, *Nuova Cronica*, ed. Giuseppe Porta, 3 vols, Parma: Fondazione Pietro Bembo; Ugo Guanda Editore, 1990–91.
- Whittington, Karl, *Body-Worlds: Opicinus de Canistris and the Medieval Cartographic Imagination*, Toronto: PIMS, 2014.
- Witt, Ronald G., 'What Did Giovannino Read and Write? Literacy in Early Renaissance Florence', *I Tatti studies*, 6, 1995, pp. 83–114.
- Zaitsev, Evgeny A., 'The Meaning of Early Medieval Geometry: From Euclid and Surveyors' Manuals to Christian Philosophy', *Isis*, 90, 1999, pp. 522–55.
- Zanna, Paolo, 'Descriptiones urbium and Elegy in Latin and Vernaculars, in the Early Middle Ages', *Studi Medievali*, Ser. 3, 32, 1991, pp. 523–96.
- Zumthor, Paul, *La Mesure du monde: Représentation de l'espace au Moyen Âge*, Paris: Seuil, 1993.

Biography: Anna Pomierny-Wąsińska — medievalist, PhD candidate at the Tadeusz Manteuffel Institute of History of the Polish Academy of Sciences. Research interests: history of culture and political history of Italian cities; late medieval historiography, urban and spatial studies, e-mail: apomierny@gmail.com.